Dynamic Competition in Networked Markets: Evidence from US Broadband

Jennifer Zou*

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Abstract

This paper examines how regulatory policies can promote sustainable entry in US broadband. I develop a dynamic spatial competition model that endogenizes market structure, product differentiation, and capacity investment. On the demand side, households differ in internet preferences by demographics; on the supply side, firms set prices and speeds in a static game subject to capacity constraints, and make dynamic expansion and upgrade decisions. To solve the firm's high-dimensional optimization problem, I develop a reinforcement learning algorithm that decomposes decisions across locations while preserving network-wide strategic coordination. In counterfactual analysis, I study two proposed policy interventions: municipal fiber provision and unbundling schemes where incumbents lease infrastructure access. Municipal broadband expands fiber access by 10% but ultimately reduces total welfare by triggering disinvestment from private competitors. This causes outsized harm to lower income households. Unbundling policies generate more favorable outcomes, particularly for price-sensitive consumers. Under a two-part tariff calibrated to the UK's cost-depreciation-based implementation, consumer surplus increases 12% at the expense of incumbents. I find that when one accounts for firms' dynamic responses, slashing connection fees while increasing usage fees by half can deliver Pareto improvements and an 18% increase in total surplus.

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1 Introduction

The US broadband market remains heavily concentrated despite rapid technological progress and rising demand for high-speed internet. Under the prevailing facilities-based competition model, each internet service provider (ISP) must build, maintain, and operate its own network. This creates substantial fixed deployment costs and makes duplicating last-mile infrastructure economically inefficient, particularly outside dense urban areas. Consequently, most local markets function as duopolies or small oligopolies with limited within-technology competition and persistent coexistence of legacy and next-generation networks.

The Federal Communications Commission (FCC) views broadband as critical infrastructure and is committed to expanding broadband equity and access through various proposed policy levers. Yet the competitive effects of infrastructure investment are complex: economies of density and scale, first-mover advantages, and spatial spillovers can reinforce incumbency and complicate efforts to promote durable competition. At the same time, bandwidth-intensive uses—such as video conferencing, streaming, and cloud synchronization—continue to grow, making it essential that policy incentivize technological innovation so that supply keeps pace with demand.

This paper develops a dynamic spatial competition model to examine how policy can foster sustainable broadband competition while preserving incentives for network expansion and quality upgrades. The model endogenizes (i) market structure through entry and expansion decisions; (ii) product differentiation via pricing and speed choices; and (iii) innovation as measured by capacity upgrades which enable higher quality product offerings. Crucially, it captures explicit network interactions across locations—adding a new line in one market affects the expansion costs into adjacent neighborhoods. These network interactions create a combinatorial problem that grows exponentially with the number of locations and decision variables, quickly becoming computationally intractable using standard methods. To address this challenge, I develop a reinforcement learning (RL) algorithm that scales to handle the high-dimensional state and action spaces. This full solution approach also enables computation of new equilibria in counterfactual policy regimes, and can be readily extended to other network environments.

The empirical analysis centers on the Boston metropolitan area from 2015 to 2019, a period featuring both rapid fiber network expansion and entry of a regional provider—ideal conditions for observing how firms reposition and invest as markets evolve. I document three key stylized facts that guide the modeling approach. First, providers respond strategically to market characteristics—entry increases with population density and internet speeds grow with income across census tracts. Second, entry becomes less likely when same-technology rivals are already present, consistent with entry deterrence motives. Third, holding tract characteristics fixed, entry is associated with a \$1.29 decrease in average plan prices and a 77.6 Mbps increase in average download speeds. These patterns highlight the different incentive channels and strategic levers that affect both static and dynamic competition in the market.

For estimation, I construct a bi-annual panel of plan menus with census tract-level availability using FCC datasets. I then recover plan-level market shares by applying machine learning techniques—specifically random forest classification and nearest neighbors clustering—to millions of consumer-initiated speed tests. This approach allows me to address a longstanding data limitation in telecommunications research. On the supply side, I infer expansion and upgrade decisions from providers' self-reported availability and maximum supported speeds. To my knowledge, this represents the first broadband demand model that allows substitution between providers and technologies at the plan-by-tract level, improving identification of key demand primitives—price and speed elasticities—and pinning down firm incentives on the supply side.

In each period, incumbent firms set plan prices and speeds subject to network capacity constraints, while households make subscription decisions each period. Households differ by income and family composition in their preferences for price and speed, and across cohorts in baseline bandwidth needs. Firms differ by technology, unobserved plan quality (congestion, reliability, etc.), infrastructure capacity, and operating costs. The demand analysis reveals intuitive but strategically important patterns: higher-income households exhibit lower price sensitivity but greater responsiveness to service quality, while larger families prioritize speed over price. Crucially, consumers prove quite elastic overall, with median elasticities suggesting slightly greater sensitivity to speed improvements than price changes. This finding has implications for competitive strategy—firms competing on quality may capture market share more effectively than those competing primarily on price, though effectiveness depends critically on local demographics.

Next, forward-looking firms decide where to expand and whether to invest in network capacity to support higher speeds across all active markets. Expansion costs vary by technology and geography; firm policy functions depend on local demographics and terrain, existing competition (including same-technology rivals that motivate preemptive entry), and expansion effects on rivals' future profits. Investment deterministically unlocks higher speed tiers through hardware upgrades; firm policy functions are shaped by current profitability, excess capacity, and competitor speed offerings. To address the previously identified computational challenges, the reinforcement learning algorithm decomposes entry decisions across locations while maintaining network-wide coordination and communication through a modified value function representation. Estimated per-kilometer expansion costs prove similar across technologies—at \$56k and \$62k for fiber and cable, respectively—and fall within industry benchmark ranges; I also find that fiber technology's operating and upgrade costs exceed those of cable.

Before delving into policy analysis, I use the model to decompose the effects of entry competition on prices, quality, and access by considering a counterfactual ban on same-technology competition. This analysis reveals that same-technology entry accelerates incumbent fiber expansion by 11% (approximately 10,700 additional households gain access each period), which is consistent with early-mover advantages. Simultaneously, competitive pressure on speed spurs quality upgrades by the cable incumbent. Entry increases consumer surplus by

19% through higher speeds, slightly lower prices, and, predominantly, increased access and plan variety—in fact, the benefit from these dynamic outcomes is an order of magnitude larger than that of static price and speed responses. Total surplus increases by 38%, suggesting that the business stealing effects from increased entry are outweighed by the benefits to consumers.

Next, I turn to two proposed policies for promoting entry. Municipal fiber provision, in which a local government owns and operates its own broadband infrastructure, presents a different competitive dynamic. The city can be thought of as a social planner that seeks to maximize total welfare. I find that the socially optimal rate of expansion is faster than that observed under private provision. However, because the municipality does not fully internalize the returns from upgrading, the planner trades off increased access for a subset of households against lower investment and subsequently lower average plan quality for all consumers. Counterintuitively, while consumer surplus from fiber increases, the total consumer surplus across the entire market actually decreases due to cable competitors' strategic responses. The dominant cable provider cuts investment in infrastructure upgrades by 10% and reduces offered plan speeds by 11%. Decomposition of the distributional effects suggests that municipal fiber benefits high-income consumers at the significant expense of low-income households. These results illustrate the challenges and potential pitfalls of introducing a new technology to market without considering the impact on consumers of the legacy technology.

Finally, I evaluate the impact of unbundling policies that decouple service competition from network ownership. Two-part tariffs, the most commonly observed fee structure, typically include a one-time connection fee to obtain access to a line and monthly leasing fees for continued use. Results suggest that under connection and leasing fees of $120 and $100, respectively (calibrated based on existing implementation in the UK), entrants and consumers benefit substantially—by taking advantage of incumbents' infrastructure to access more households, entrant profits increase 58% and consumer surplus grows by 12%—while incumbent profits decline. This highlights one of the downsides to cost-depreciation-based fees, which do not account for firms' dynamic, strategic behavior. I show that alternative tariff structures can promote entry and improve consumer welfare while maintaining incumbents' baseline profits. For instance, dropping connection fees to \$15 (thereby significantly reducing sunk costs for potential entrants) while increasing usage fees to \$160 (increasing per-period transfers to incumbents) allows all firms' profits to grow relative to the status quo. Under this approach, consumer surplus grows by 14% and subscribers of both cable and fiber are better off, with low-income households benefiting in particular from the changes in fiber.

Related Literature

This paper contributes to several research areas. Most directly, it extends the literature on broadband competition, which has historically examined either demand estimation or supply-side behavior in isolation due to data limitations. Goetz (2019) embeds internet demand models into ISP-content provider bargaining frameworks but relies on aggregate county-level firm shares. Nevo et al. (2016) estimate residential broadband demand using high-frequency customer data from a single firm, providing rich insights into consumer heterogeneity but limited ability to analyze competitive interactions across providers. By leveraging machine learning techniques, this analysis captures both consumer heterogeneity and inter-firm substitution patterns with improved granularity.

More broadly, this work builds on the established literature studying competition in telecommunications markets. Greenstein and Mazzeo (2006) examine product differentiation among competitive local exchange carriers, finding that product heterogeneity predicts entry decisions and market structure evolution. Fan (2013) analyzes how subsidies affect US telephone market entry through a dynamic structural model that captures heterogeneous option values, while Wilson et al. (2021) investigates how entry threats shape incumbent firm decisions and long-run market structure. My model extends this work by endogenizing the relationships between market structure, product differentiation, and firm investment decisions. Seamans (2012), Landgraf (2020), and Gillett et al. (2006) examine specific cases of government entry into telecommunications markets; here, I extend their work by explicitly modeling the strategic responses of private firms to public entry and quantifying the welfare effects of different institutional arrangements.

This paper also contributes to the competitive dynamics literature and, specifically, to empirical work on competitive interaction in networked markets. Chen and Miller (2012, 2015) synthesize prior research around three factors—market characteristics, attacker/entrant characteristics, and defender/incumbent characteristics. I build a flexible framework that allows for a broad host of strategic responses (pricing and quality differentiation, capacity upgrades, and entry timing) to each factor, while making spatial interdependencies across local markets explicit. Related empirical work shows how spatial and cross-market linkages shape conduct and investment: in retail/franchising, firms balance encroachment/cannibalization against density economies and path dependence in rollouts (Kalniņš (2004); Bronnenberg and Mela (2004); Holmes (2011); also Seim (2006); Mazzeo (2002); Ellickson (2007)), and in airlines, multimarket contact and network position structure rivalry and responses (Evans and Kessides (1994); Gimeno (1999); Aguirregabiria and Ho (2012); Ciliberto and Tamer (2009); Ciliberto and Williams (2014)). Extending this, I formalize how market linkages and network position—emphasized by recent strategy work on entry threats (Ethiraj and Zhou (2019))—generate heterogeneity in competitive responses both within the same incumbent across markets and across firms.

Finally, this paper makes methodological contributions to the dynamic games literature in industrial organization. Popular approaches such as those developed by Hotz and Miller (1993), Bajari et al. (2007), and Pakes et al. (2007) avoid solving for full equilibrium to maintain computational tractability, but still struggle with the high-dimensional state and action spaces that arise naturally in network industries. The reinforcement learning approach developed here handles these computational challenges, maintains the essential network relationships and strategic interactions, and solves for full equilibria. The methodology builds on earlier work on stochastic algorithms by Pakes and McGuire (2001) and Fershtman and Pakes (2012). More recently, related work by Sweeting (2013) and Collard-Wexler (2013) has proposed full solution methods in other dynamic settings. However, broadband's spatial network structure creates unique computational hurdles that motivate the algorithmic innovations presented in this paper. The broader methodology also connects to emerging applications of machine learning techniques in industrial organization (e.g., Bajari et al. (2015); Gentzkow et al. (2019)). In particular, the clustering approach developed here to infer plan-level market shares from consumer speed-test data addresses a longstanding measurement bottleneck in telecommunications research and may prove applicable in other settings where direct choice data are unavailable.

The remainder of the paper proceeds as follows. Section 2 provides industry background, describes data construction, and presents descriptive evidence motivating the structural model. Section 3 presents the dynamic model of household demand and firm behavior. Section 4 details the reinforcement learning estimation strategy. Section 5 discusses empirical results, and Section 6 analyzes counterfactual policies including municipal broadband and unbundling schemes. Section 7 concludes with broader implications for competition policy in network industries.

2 Industry Setting and Data

The US broadband market's highly concentrated structure can be traced back to the breakup of AT&T in 1984, which divided the telecommunications giant into seven "Baby Bells." These entities inherited control over separate regional telephone networks, creating a legacy of locally monopolized markets that persists today. In recent decades, the Federal Communications Commission, recognizing broadband as critical infrastructure essential for public safety, health, education, and the economy, has significantly increased public involvement and funding in the market. The 2021 Infrastructure Investment and Jobs Act, for example, allocated \$42.5 billion for states to expand broadband equity, access, and deployment. Despite this increased public investment, access and provision remain predominantly handled by private firms driven by private interests, with limited exceptions such as municipal broadband programs.

The focus of broadband competition centers on the ”last mile”—the provision of internet services directly to consumers[1]. The facilities-based competition model that governs the last mile requires each internet service provider (ISP) to build, maintain, and operate its own infrastructure. While this approach encourages private investment in network development, it creates substantial barriers to entry due to high fixed costs. Market entry requires providers to navigate a complex process: they must apply for local permits, construct central hubs where transmission hardware resides, and then deploy extensive networks of cables to physically connect every household. Infrastructure deployment involves significant logistical challenges, including digging up roads and sidewalks or obtaining access to electricity poles for aerial installation. These high costs—primarily material and labor expenses—create barriers that naturally limit competition. To maximize efficiency and minimize repeat installation costs, ISPs must be forward-looking: they install cables that include both active wires for current data transmission and dark fiber for anticipated future demand.

Providers are differentiated by access technology and internet service quality. Quality is characterized by several key measures. Speed or bandwidth determines how much data can be transferred and is roughly proportional to the number of active wires. Latency and reliability are less commonly advertised but equally important for utility; the former measures the time required for data transmission while the latter encompasses congestion issues—where excessive data transfer across the network leads to increased latency—and outages caused by infrastructure failures that prevent data transmission entirely. Individual firms typically offer multiple service tiers with different speeds at varying price points, controlling quality through routing protocols and network management.

[1] The transmission of data between competitors’ networks occurs upstream, and any bargaining between firms over transmission fees is generally negligible and taken as given in competitive analysis.

The technological evolution of broadband delivery has also shaped market dynamics. The late 1990s and early 2000s witnessed the emergence of DSL and cable broadband technologies, which leveraged existing infrastructure: DSL utilized the Baby Bells' copper telephone lines, while cable providers upgraded their coaxial television networks. Each technology has also experienced generational improvements to speeds through the introduction of new transmission hardware and protocols (such as DOCSIS for cable systems). In the 2010s, declining costs made fiber-to-the-home (FTTH) deployment increasingly viable as a competitive alternative.

Fiber technology offers several critical advantages over legacy systems, including symmetric speeds with equal upload and download rates, greater reliability, and superior scalability that allows networks to meet rapidly growing demand for bandwidth-intensive applications[2]. The asymmetry between download and upload speeds in traditional cable systems has become increasingly problematic as consumer behavior has evolved. While early internet usage was dominated by downloading content such as web pages and streaming video, contemporary demand increasingly requires substantial upload capacity for activities like video conferencing, cloud storage synchronization, and content creation. This shift makes fiber's symmetric capabilities particularly appealing and positions it as a "future-proof" broadband technology.

However, despite these technological advantages, access to fiber remains limited in the US—only 43% of households are serviced by a fiber provider, compared to 89% who have access to cable internet service[3]. Fiber deployment is especially expensive (because it cannot rely on repurposing existing infrastructure) and is influenced by several market factors including geographic terrain, household density, and consumer demographics. The economic inefficiencies associated with building duplicate network infrastructure make it such that competition occurs primarily across different technology types rather than within them.

[2] According to a 2024 report by the consulting firm Deloitte.

[3] According to a 2022 report by Fierce Network, which provides 'news and analysis [on the] ... global communications industry'

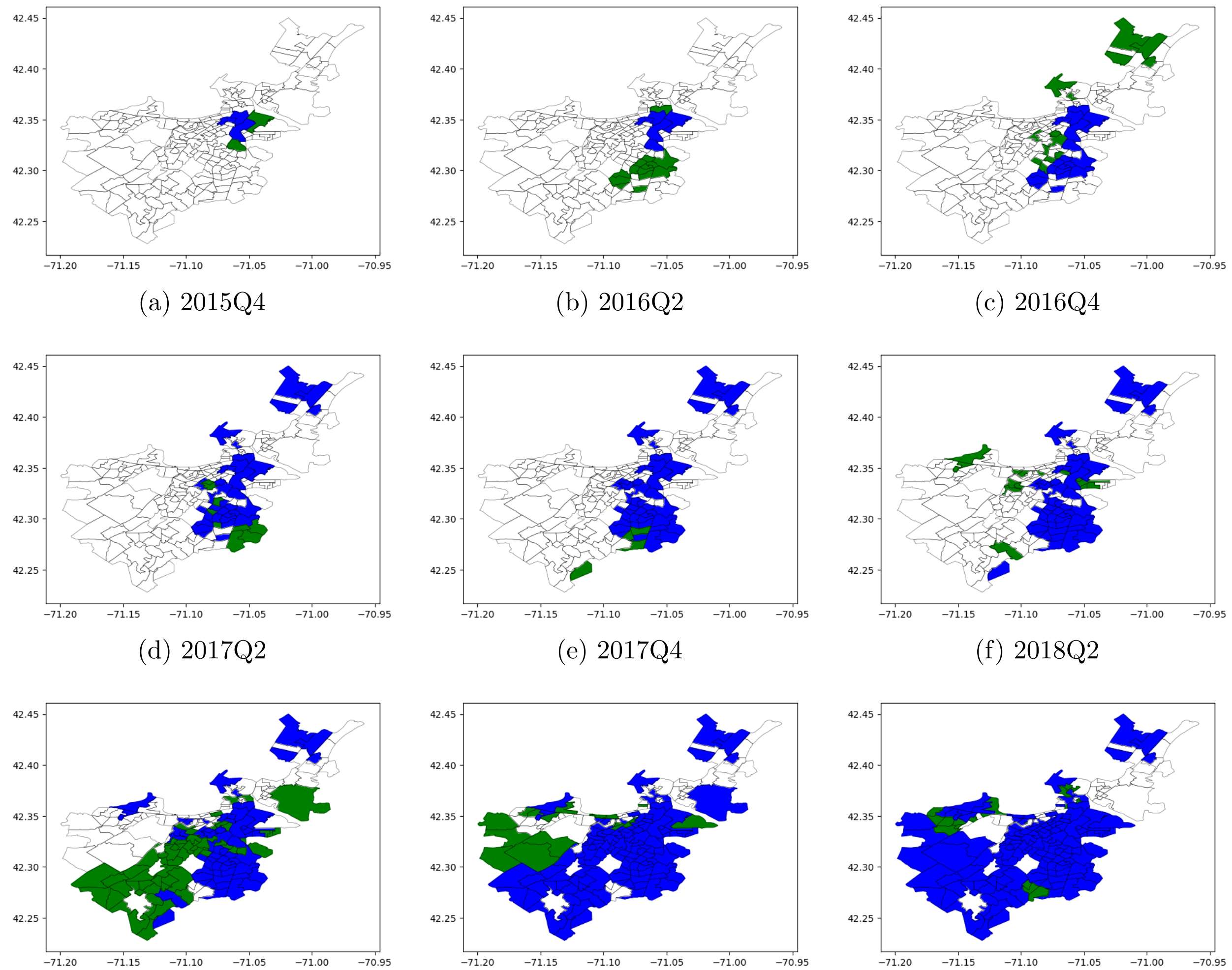
While legacy technologies[4] continue to maintain market presence, this paper focuses on competition between cable and fiber providers.

The Boston metropolitan area from 2015 to 2019 provides an ideal setting for analyzing broadband competition. Cable technology, provided primarily by Comcast, represents the predominant broadband option and is available across all market segments. Most households also have access to some combination of legacy DSL and satellite services. During the window of 2015 to 2019, the region experienced the entry and rapid expansion of its first fiber provider, Verizon, as well as the entry of a single regional provider, RCN, which offered both fiber and cable services[5]. Figure 1 illustrates the expansion path of Verizon's fiber network over this time frame. This dynamic environment enables me to analyze the impact of same-technology entry on competition, identify differences in cable and fiber expansion strategies, and quantify incentives for further infrastructure investment and capacity upgrades, among other competitive channels.

Various policy interventions have been proposed and enacted to address broadband market concentration. The 21st century has witnessed two waves of municipal broadband provision, where towns offer internet services over their own electric utility networks. However, due to lobbying by telecommunications firms, including Comcast and Verizon, more than 15 states have passed laws prohibiting municipal broadband. In 2016, under Title II of the Telecommunications Act, the FCC considered mandating the unbundling of broadband infrastructure to make internet service provision more competitive. Unbundling would require incumbent network owners to lease parts of their infrastructure to competitors. The degree of access can vary, from direct access to copper or fiber lines (local loop unbundling, LLU) to wholesale broadband access over the incumbent's equipment (bitstream access),

[4]These include DSL and wireless options, such as satellite and fixed wireless, which typically offer much lower speeds and see limited adoption in areas where cable and fiber are available.

[5]RCN is what is known as an 'overbuilder', a provider that minimizes expansion costs by following existing road/utility infrastructure. As such, they are able to compete in technologies where other firms cannot, but are also limited in where they are able to expand. This behavior is accounted for in the model through a restricted action space and different expansion costs. I treat RCN fiber and RCN cable as two separate firms for ease of estimation; in fact, there does not visually seem to be much coordination in expansion behavior between the two.



(a) 2015Q4 (b) 2016Q2 (c) 2016Q4

(d) 2017Q2 (e) 2017Q4 (f) 2018Q2

(g) 2018Q4 (h) 2019Q2 (i) 2019Q4

Figure 1: Expansion of Verizon fiber services across Boston census tracts. Tracts are shaded blue if services were offered in the previous period and green for new expansion of services.

with competitors able to differentiate at higher service layers. This paper studies the entry of municipal fiber—where the town acts as a social planner seeking to maximize total welfare from the technology—and the equilibrium impact of different LLU policies.

Globally, the variety of unbundling policies highlights the difficulties of design and implementation. The European Union, for example, mandated unbundling in 2000 under the Unbundled Access Regulation. National regulators (such as Ofcom in the UK and BNetzA in Germany) set one-time connection fees and monthly rental charges based on the incumbent's costs, including capital depreciation and a reasonable rate of return. Even within this framework, different cost models and ambiguity around what constitutes ”reasonable” returns complicate implementation. In Japan, fees are proposed by the incumbent and approved by the Ministry of Internal Affairs and Communications based on bottom-up cost models. South Korea focused on bitstream access rather than direct access to physical infrastructure, with fees set through negotiation under oversight by the Korea Communications Commission. This paper evaluates whether an optimal tariff structure exists that represents a Pareto improvement over the status quo—that is, whether a LLU policy can enable new entry and improve consumer outcomes through increased competition while ensuring the same level of incumbent profits.

2.1 Data

This analysis uses a bi-annual panel dataset of broadband plan characteristics and market shares from 2015 to 2019. I construct this dataset by combining information from multiple sources. Service availability data come primarily from the FCC's Form 477, which is collected from ISPs at a bi-annual frequency for every technology they offer. This mandatory reporting includes the maximum speeds supported by each provider's network in every census block, allowing for identification of network upgrade and investment decisions through discrete jumps in reported speeds.

Plan characteristics are sourced from the FCC's Urban Rate Survey, which provides speeds (upload and download) and prices for the menu of plans offered by ISPs in a random sample of urban areas each year. To fill gaps in coverage, I supplement this with historical ISP website data sampled biannually from the Internet Archive's Wayback Machine. A few additional assumptions are required to overcome limitations to the available data. First, I assume that national providers advertise the same price for a given speed across all active markets[6]. Furthermore, promotional discounts on advertised prices, which are not observed

[6] This is necessary because the Wayback Machine does not distinguish between locations in its archives. In other words, the same URL may correspond to saved pages showing products available in Boston markets on one date and Philadelphia on another.

but are frequently offered in broadband, are applied equally across all plans in a market on average. Finally, in any given period, a provider offers the same set of speeds across all markets; any variation in plan availability arises from network capacity constraints[7]. I join these plan characteristics with the Form 477 availability data to construct consumer choice sets in each census tract.

Market shares at the plan level are computed from several complementary sources. Core subscriber data comes from Capital IQ Pro, which provides quarterly ISP-level subscriber counts by zip code. To estimate plan-level shares within each ISP, I combine three data sources: Measurement Lab speed test data, which contains ISP, timestamp, location, and measured speeds for millions of user-initiated tests; the FCC's Measuring Broadband America program, which provides annual household-level data on plan choices and measured speeds from a random sample of households; and actual monthly plan shares from an anonymous ISP covering 4 metropolitan statistical areas (MSAs) for a period of 10 months in 2016. The speed test data may suffer from selection bias as a bulk of tests are initiated when consumers experience service issues. However, under the assumption that these outages affect all plan tiers equally, the relative share distribution should be preserved in the remaining 'normal' tests. I train a random forest classifier and a nearest neighbors clustering algorithm to map speed tests to plan choices[8]. Appendix B outlines the steps of the clustering process and the robustness checks taken to prevent overfitting. Figure 28 demonstrates that the model fits the actual shares of the anonymous ISP fairly well.

Finally, consumer demographic information comes from the Census's Annual Community Survey. Of particular interest is the joint distribution of income, marriage status, and presence of children, which all plausibly affect household willingness to pay for higher speeds.

These data sources report information at different geographic granularities and time

[7]Again, due to Wayback Machine data limitations. This assumption, though, is largely supported by patterns observed in the Urban Rate Study dataset.

[8]The random forest classifier identifies anomalous tests and the access technology type for subscribers of multi-tech providers. The nearest neighbors algorithm clusters tests around the reported availability by technology type.

intervals. I define a market to be the smallest area—the census tract—in a 6-month period and use residential-weighted ratios to map all datasets to this geography. In the broadband industry, there can be substantial variation in provider availability from one street to the next, so this market definition allows me to best approximate households' true choice sets while maintaining computational tractability.

2.2 Descriptive Statistics and Stylized Facts

Table 1 summarizes select tract-level characteristics. The average market includes just under 800 households with a median income of approximately $120k, however, there is substantial variation even across tracts within Boston. A consumer in the average market has a choice between 5 providers offering 18 different plans across 4 technology types. This validates that most of the competition occurs across technologies rather than within.

Table 2 summarizes firm- and plan-level characteristics. The average bandwidth capacity of a firm is around 500 megabits per second (Mbps). This varies significantly across firms and especially across technology types (e.g. satellite can have a minimum of 15 Mbps while fiber reaches up to 1000 Mbps). In a market, the average firm offers between 2 and 3 plans and has around 11.4% market share. Plans average 110 Mbps of advertised download and 31 Mbps upload speeds at a price of $76 per month. The average plan commands a market share of 6.7%.

Next, I document three facts about the market that motivate future modeling decisions. First, I find that firm strategy responds to market characteristics. Figure 2 compares internet access (as measured by the number of available providers in a given market) on the left against population density[9] on the right. The plots suggest, quite intuitively, that firms are more likely to enter markets where the build distances and therefore per-household costs are lower. The dark central region corresponds to the downtown area of Boston. However, access does not necessarily equal quality. Figure 3 plots the average advertised download

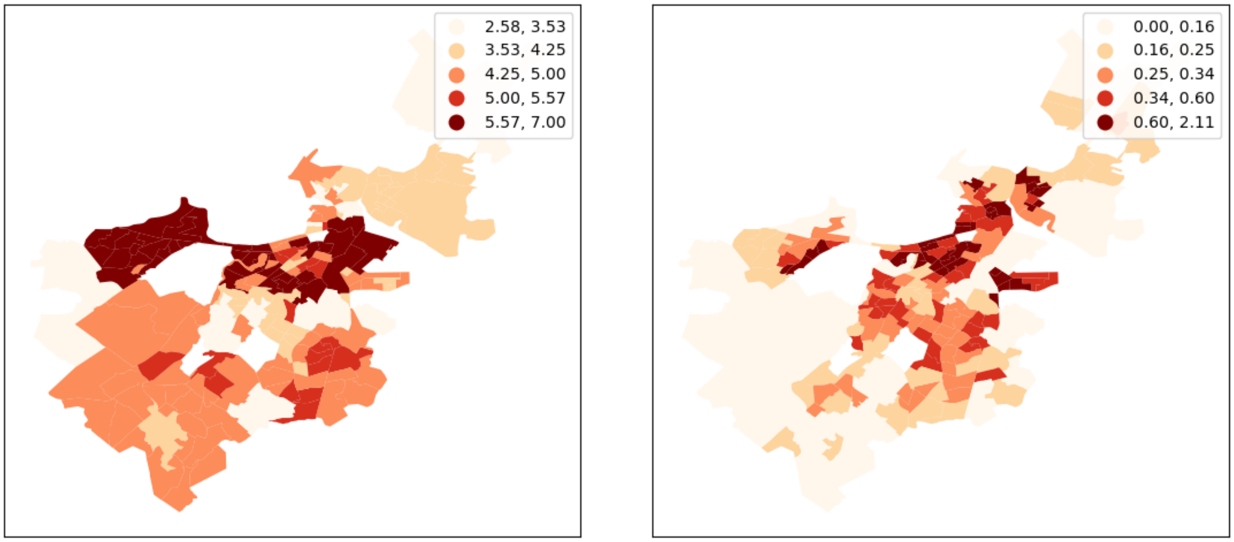
[9]Population itself is not an informative metric because census tracts are drawn to have roughly the same number of households.

Variable	Unit	Mean	Standard Deviation	Minimum	Maximum
Population	integer	797.214	430.606	3.000	2417.000
Median Income	\$1,000	117.654	61.100	20.108	350.000
Married	%	61.982	21.345	0.000	1.000
Has Kids	%	40.487	13.694	0.000	1.000
ISPs	integer	5.443	1.451	1.000	9.000
Technologies	integer	3.610	1.050	1.000	5.000
Plans	integer	18.399	5.719	5.000	41.000

Table 1: Description of Tract-level Data.

Variable	Unit	Mean	Standard Deviation	Minimum	Maximum
Max Bandwidth	Mbps	471.932	418.122	15.000	1000.000
Menu Size	integer	2.429	1.397	1.000	5.000
Avg. Market Share	%	11.448	15.621	1.132	45.385
Download	Mbps	109.367	120.752	1.000	500.000
Upload	Mbps	30.762	86.030	0.384	500.000
Price	\$	76.44	31.60	19.99	144.99
Avg. Market Share	%	6.749	6.775	1.057	34.238

Table 2: Description of Firm- and Plan-level Data.



(a) Entry

(b) Density

Figure 2: Heterogeneity across markets: number of firms (left) vs population density (right).

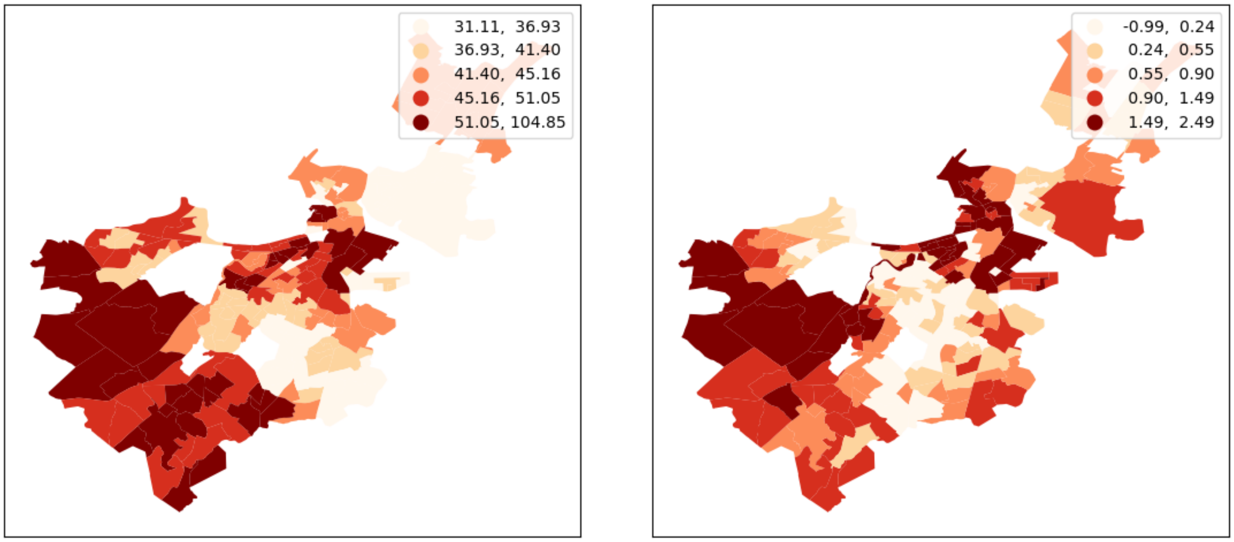
speeds on the left and the average log household income on the right. These panels indicate that firms are more likely to offer faster plan tiers in markets where household income is higher, regardless of market density. Furthermore, this suggests that incentives to invest in capacity and quality are steered by heterogeneous consumer preferences in addition to firm competition. The former channel can be seen in the western tracts with low density and low firm entry, but extremely high average speeds; the latter is evidenced by the tracts just west of downtown, where household income is low but entry and offered speeds are high.

Next, I present evidence suggestive of first-mover benefits or entry deterrence incentives for fiber providers[10]. To quantify the influence of a same-technology competitor on entry decisions, I run a logistic regression with the following specification:

$$entry_{mft} \sim same\_tech_{mft-1} + \beta x_{mt} + d_t + d_f$$

where *entry* is an indicator for whether firm f entered market m in period t. *same_tech* is

[10] I'm unable to determine if the same holds true of cable providers as the incumbent, Comcast, already operates in every market in my dataset.



(a) Download Speeds

(b) Income

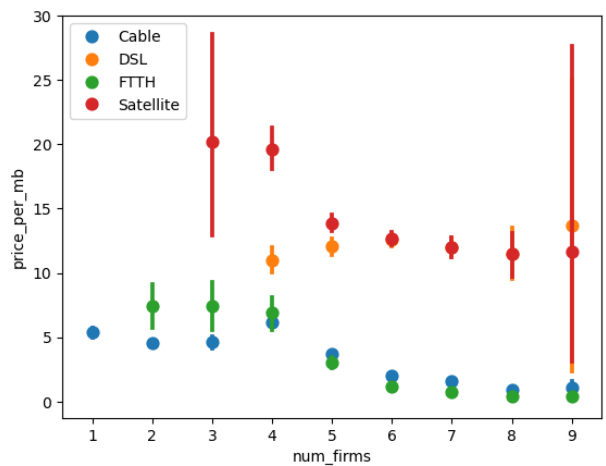
Figure 3: Heterogeneity across markets: plan speeds (left) vs log(income) (right).

	(1)	(2)	(3)
same_tech	-0.0495***	-0.0461***	-0.0257*
	(0.010)	(0.010)	(0.014)
Period FE	no	yes	yes
Firm FE	no	no	yes
R^2	0.019	0.073	0.109

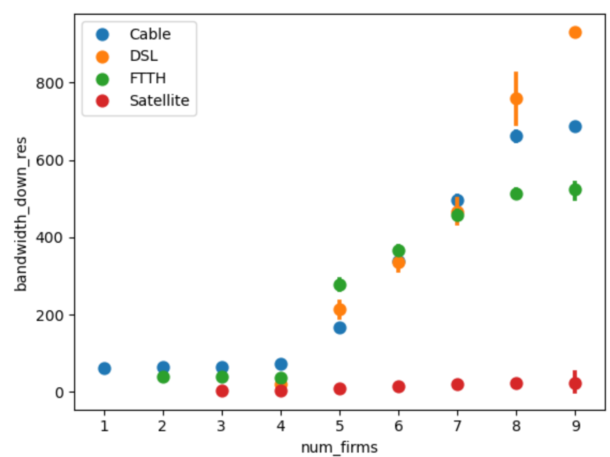
Table 3: Impact of same technology competition on entry, controlling for observable market characteristics.

an indicator for whether a provider of the same technology type is active the market, x_{mt} are other observable market characteristics (such as population density, household income, etc.), and d are various fixed effects. Table 3 shows the regression results. I find that the presence of a same-technology competitor leads to a small but significant 2.6 to 5.0% decline in the likelihood of the firm expanding into the same market.

Finally, I find that increased competition leads to more desirable plans for consumers. Figure 4 shows the relationship between the number of firms operating in a market and the average prices and speeds of the offered plans. On the left, the price per megabit of download speed decreases consistently with increased entry after the number of firms reaches at least 3. On the right, the advertised download speed—residualized by a time trend to account for faster speeds over time across the industry as a whole—increases monotonically with the number of firms. These trends are consistent across the major technology types, cable and fiber, and present to a lesser degree for the legacy technologies, DSL and satellite. Interestingly, I only observe the entry of cable and fiber competitors in my data, which anecdotally suggests that same-type entry may be a stronger competitive factor than general entry.



(a) Prices



(b) Speeds

Figure 4: Relationship between number of firms in a market and the average price per MB (left) and download speed (right); bars represent 95% CIs.

To quantify the impact of entry on competition in prices and speeds, I run the following regression:

$$\Delta \bar{y}_{m,t+\Delta t} \sim \beta_0 entry_{mt} + \beta_1 num\_tech_{mt} + \beta_2 num\_provider_{mt} + d_t$$

where y is the outcome of interest and Δt is a time lag to allow for competitor responses. I include variables capturing observable market structure, such as the number of existing providers and technologies, as well as period fixed effects d_t, to account for general market trends over time. The entry decision $entry_{mt}$ is almost certainly endogenous and correlated with outcomes Δy. To account for this, I instrument the decision with the number of firms operating in neighboring markets in the previous period, a proxy for the number of potential

	OLS		IV	
	t+1	t+2	t+1	t+2
entry	-3.7729***	-5.4818***	-1.2858*	-2.6745***
	(0.374)	(0.461)	(0.7030)	(0.8447)
num_tech	0.7157***	0.5671**	1.3069	2.0441*
	(0.198)	(0.244)	(0.9959)	(1.0963)
num_isp	-0.7001***	-0.8958***	-0.3365	-0.2758
	(0.166)	(0.205)	(0.6414)	(0.7072)
R^2	0.484	0.414	0.3101	0.5663
F-stat	—	—	51.49	51.49

Table 4: Impact of entry on prices in the 6 months (t+1) and 12 months (t+2) following.

entrants. Under the assumption that expansion is slow but price and speed setting can occur instantaneously, this affects the likelihood of entry independently of plan characteristics or market profitability in the current period. The first-stage F-statistic suggests this is a sufficient instrument for entry.

Tables 4 and 5 show the regressions for price and speed outcomes, respectively. The first two columns include OLS results for 1- and 2-period lags. I find that every additional firm that enters leads to a $3.77 decline in monthly prices in the subsequent period; this effect grows to $5.48 one year after entry. Entry also leads to a 77.6 Mbps and 56.9 Mbps jump in advertised download speeds in the following one and two periods, respectively. IV estimates can be found in third and fourth columns. I find that the results are consistent in sign with the OLS results for all outcomes, but smaller in magnitude, although still statistically significant. The trend across time lags also holds. These results suggest that, holding all else fixed, policies that incentivize additional entry into this highly concentrated market can be beneficial for consumers. However, these results are just suggestive—to accurately estimate the consumer welfare implications, I turn to the structural model to provide a more comprehensive, long-run perspective.

	OLS		IV	
	t+1	t+2	t+1	t+2
entry	77.6175***	56.8847***	31.897*	43.454*
	(11.716)	(14.904)	(17.879)	(25.064)
num_tech	-10.3594*	-18.0844**	7.8944	16.063
	(6.200)	(7.887)	(23.957)	(33.328)
num_isp	1.6100	8.5573	2.5412	6.0032
	(5.199)	(6.614)	(15.386)	(21.437)
R^2	0.153	0.228	0.1374	0.2607
F-stat	—	—	51.49	51.49

Table 5: Impact of entry on download speeds in the 6 months (t+1) and 12 months (t+2) following.

3 Model

In this section, I introduce a dynamic model of firm entry and investment. There are two types of entry decisions a broadband firm undertakes: 1) the decision to set up a regional hub (i.e. entry into the broader metro area), and 2) the decisions to connect each census tract to the hub through an existing network (i.e. expansion). I observe both types of entry in the data; however, the costs of the former are not identified without further assumptions on (unobserved) potential entrant behavior. Even if these costs were identified, it is computationally infeasible to evaluate every combination of initial markets to service (the number of choices is on the order of 2^M where $M = 208$ census tracts in the case of Boston). Although I include the possibility of a new potential entrants in the framework as they are relevant to the counterfactual policies of interest[11], in estimation I take the initial entry decision as given and focus instead on the firms' expansion strategy. Going forward, entry and expansion interchangeably refer to the decision to connect a census tract to existing infrastructure unless otherwise noted. Exit is not allowed and very uncommon in the broadband market.

Embedded within this dynamic framework, incumbent firms engage in a static, Bertrand game of price and speed competition. Myopic households have heterogeneous preferences for

[11]Fortunately, non-identification of entry costs does not significantly affect the analysis as, under unbundling, any entirely new potential entrants would not have to pay these hub-related sunk costs anyways.

internet and make plan choices every period. Time is discrete and each period corresponds to six months. Under this setup, I can evaluate the short-run effects of entry and competition on incumbent profits through consumer preferences in each market and how this changes long-run expansion and investment strategies in equilibrium. The precise timing of the model is as follows:

1. Incumbents play a stage game in which they compete on plan speeds, then prices separately in each tract.

2. Incumbents observe their private expansion cost shocks and make expansion decisions in every adjacent tract. They also observe a private investment cost shock and make a single investment decision that affects all of their active markets[12].

3. One new potential entrant observes its private entry cost shocks and makes a regional entry decision. If it does not enter, it dies.

4. Households choose internet plans from incumbent offerings; firm profits are realized.

5. Infrastructure quality evolves deterministically with investment decisions and the state of the market transitions with entry and expansion.

In the first half of this section, I describe the household demand model and the firms' static profit maximization problem. In the second half, I present the dynamic objective that governs firms' expansion and investment decisions.

3.1 Internet Demand

In census tract m, each household i in period t is characterized by three observed demographic characteristics y_{it}: marriage status and whether the household has children (which

[12] In reality, firms can improve their network capacity in two ways: upgrading the software and hardware at their regional hub, a single action that affects their entire network, or installing new cables to reroute traffic, a localized action that affects a subset of markets. Because the latter is extremely costly (in fact, firms actively take steps to reduce the need for reinstallation) and I do not observe where cables are physically installed, I omit this type of investment from the model.

collectively serve as a proxy for household size), and income. These demographics are drawn from a location- and time-specific joint distribution denoted F^m_{yt}. All households within tract m have access to every plan offered by firms in that market. Households i choose internet plans j every period t to maximize their indirect utility given by

$$u_{ijt} = \underbrace{\beta_p p_{jt} + \beta_b b_{jt} + x'_{jt}\beta + \xi_{jt}}_{\delta_{jt}} + \underbrace{\sum_{k=p,b}\sum_{y} k_{jt} y_{it} \sigma^y_k}_{\mu_{ijt}} + \epsilon_{ijt}$$

where p is the monthly charge (in \$100), b is the (log) advertised maximum download speed, and x include other plan characteristics such as upload speed and data usage caps, as well as technology-type, year, and firm fixed effects. ξ is the unobserved demand shock, which can be interpreted as a plan's service quality or reliability. y includes two transformations of income: log monthly income and log monthly income squared. $\sigma^y_p, \sigma^y_b \in \mathbb{R}^3$ capture consumer heterogeneity in their preferences for price and speed, respectively, according to observable demographics. ϵ are iid type-I extreme value shocks. δ_0 is normalized to 0 for the outside option of selecting no internet plan.

Let $\theta = (\beta_p, \beta_b, \beta, \sigma^y_p, \sigma^y_b)$ denote the vector of demand parameters. Under the logit assumption, market shares for plan j are given by the following expression:

$$s_{jt}(\theta) = \int \frac{\exp(\delta_{jt} + \mu_{ijt})}{1 + \sum_l \exp(\delta_{lt} + \mu_{ilt})} dF_y(y_{it})$$

Although choice sets vary at the census tract level, observed plan shares in my data are reported by zip code and there does not exist a 1:1 mapping between the two geographies. For estimation purposes, I infer the tract shares by taking the residential-weighted sum of zip shares:

$$s^m_{jt} = \sum_z w^m_{zt} s^z_{jt} \quad ; \quad \sum_z w^m_{zt} = 1 \; \forall m$$

where w_{zt}^m is the share of households in tract m that also belong to zip z and s_{jt}^z is the observed zip-level plan share obtained from scaling ISP shares by the conditional plan shares estimated from the speed test data, $\hat{s}_{j|f}$.

The household demand model pins down the firm's profit incentives; next, I will discuss how the firm chooses its menu of plans to offer in order to maximize said profits in each period.

3.2 Plan Prices and Speeds

In the stage game, I assume that incumbent firms compete sequentially on speeds and then prices separately in each tract[13]. Working backwards in time, ISP f sets prices for its plans in a game of Bertrand competition in the second subperiod:

$$p_{ft}^{m*} = \arg\max_{p_{ft}} \pi_t^m(p_{ft}, b_{ft}) = 6 \cdot \sum_j (p_{jt} - c_{f,0}^m - c_{ft,1} b_{jt}^{m*}(p_{jt})) s_{jt}^m(p_t, b_t^{m*}(p_{ft}))$$

In the first subperiod, ISP f selects speeds for a menu of up to 3 plans[14], subject to the bandwidth capacity constraint $\bar{b}$ determined by the cumulative investment in their network infrastructure[15], in order to maximize its static profits:

$$b_{ft}^{m*} = \arg\max_{b_{ft} \leq \bar{b}_{ft}^m} \pi_t^m(p_{ft}, b_{ft})$$

I assume constant marginal costs c_0 are firm- and tract-specific; these can be interpreted as customer service costs that scale with the number of subscribers. Variable marginal costs c_1 are firm- and time-specific, capturing, for example, electricity and utility costs that scale with total bandwidth usage. I also consider other specifications for marginal cost including

[13]This assumption can be thought of as the firm responding to different competition with different promotions.

[14]This restriction is motivated by tract statistics observed in the data and a 2020 article by CCG Consulting discussing the common number of tier offerings. CCG Consulting is a firm which specializes in "helping clients launch new broadband markets". https://potsandpansbyccg.com/2020/11/06/pricing-strategies/

[15]Note that b here denotes bandwidth in levels, not logs.

constant and quadratic functional forms; this linear specification best explains the prices and speeds observed in the data. Details can be found in Appendix D.

Optimal prices and speeds satisfy the following first order conditions in every market m:

$$p_{ft}^{m*} + [\Omega_{ft}^{p,m}]^{-1} \cdot s_{ft}^{m}(p_t^{m*}, b_t^{m*}) = c_{f,0}^{m} + c_{ft,1} b_{ft}^{m*}$$
$$p_{ft}^{m*} - [\Omega_{ft}^{b,m}]^{-1} \cdot (c_{ft,1} s_{ft}^{m}(p_t^{m*}, b_t^{m*}) - \lambda) = c_{f,0}^{m} + c_{ft,1} b_{ft}^{m*}$$

where p, b, and s are length-J vectors, Ω^p, Ω^q are the $J \times J$ matrices of price and speed elasticities, respectively, and λ is a Lagrange multiplier. Note that price markup $[\Omega^p]^{-1} \cdot s$ depends on the other options offered in the firm's menu—firms often use this to 'steer' consumers toward a certain plan tier[16]. The speed 'markup' is additionally scaled by the marginal cost of providing higher speeds; firms have incentive to intentionally cap speeds below maximum capacity to manage network congestion during peak hours. Going forward, profits from the static game are taken as observed in the dynamic game.

3.3 Expansion and Investment

Let $s_{ft} = (s_{ft}^1, ..., s_{ft}^M, \tilde{s}_{ft})$ denote firm f's state in period t. The state includes vectors of tract-specific characteristics $s_{ft}^m \in \mathcal{S}$, for all markets $m = 1, ..., M$ regardless of service availability:

$$s_{ft}^m = (d_{ft}^m, h(\{\xi_{jt}^m\}_j), \bar{u}_{-ft}^m, nn_t^m, same_{g(f),t}^m)$$

as well as an aggregate variable $\tilde{s}_{ft}$

$$\tilde{s}_{ft} = (\sum_m \pi_{ft}^m, cap_{ft}, \bar{b}_{ft}, \max b_{-ft})$$

[16] See, again, the article from CCG Consulting.

Here, d_{ft}^m is the distance (in kilometers) to the nearest market that firm f currently operates in—this variable is equal to -1 if f does not operate in any adjacent markets to m and 0 if f already operates in m. ξ_{jt}^m are the measures of the firm's unobserved plan quality estimated from the demand system and h is a function mapping the set of plan qualities to a one-dimensional value on $\mathbb{R}$, $\bar{u}_{-ft}^m$ is the inclusive value of all plans offered by competing firms in the market, nn_t^m is the number of firms operating in neighboring markets, and $same_{g(f),t}^m$ is an indicator for whether there exists a same-technology competitor in the market. These capture the firm's market position, the competitiveness and positioning of other firms in the market, the degree of entry threat, and entry deterrence incentives, respectively, which all influence the firm's entry/expansion decisions.

The aggregate state variable captures network-wide characteristics that inform the investment decision. These include: the firms' total plan revenue ($\sum_m \pi_{ft}^m$), the average free capacity (i.e. the difference between network capacity and the fastest speed tier in a market, cap_{ft}), the average network capacity ($\bar{b}_{ft}$), and competitors' maximum offered speed ($\max b_{-ft}$). Intuitively, the firm's investment strategy depends on the returns it can capture from upgrading and the competitive speed pressure it faces. The state of the market as a whole is given by $s_t = \{s_{1t}, ..., s_{Ft}\}$.

Once plan characteristics are set, incumbent firms observe their private action-specific cost shocks ϵ_{ft}—assumed to be i.i.d. type-I extreme value and independent over time—and make expansion and investment decisions. The firm's actions can be encoded as a binary vector of length $M+1$, $a \in \mathcal{A} = \{0,1\}^{M+1}$, where element $a_m = 1$ indicates expansion into tract m and $a_{M+1} = 1$ indicates a network upgrade was taken. Expansion decisions are restricted to a two-tract radius (that is, directly adjacent to or adjacent to an adjacent tract) around the incumbent's existing network[17]. This means that $a_m \geq 1$ only in markets satisfying $\sum_{m' \in A(\{m\}) \bigcup A(A(\{m\}))} \mathbb{1}\{d^{m'} = 0\} \geq 1$ where $A(\{m\})$ is the set of tracts neighboring

[17]For the overbuilders, the expansion neighborhood is further restricted to just the immediately adjacent markets.

$\{m\}$. Furthermore, I assume that new generations of a technology[18] are always available so that the upgrade outcome is deterministic. Moreover, I fix the available capacity tiers at the bandwidths most commonly reported in the data. These tiers are the same across all firms, which is a reasonable assumption given that transmission protocols are public knowledge.

Incumbent firm f's flow profits

$$u(s_{ft}, a; s_{-ft}) + \eta\epsilon_{ft}(a) = \pi_{ft}(s_{ft}, s_{-ft}) - c(s_{ft}, a) + \eta\epsilon_{ft}(a)$$

are the sum of stage game revenues, action-specific costs, and the firm's cost shocks across all markets. Scale parameter η captures the variance of the cost shock and can be interpreted as all other factors affecting the firm's decision not captured by π and c[19]. I assume the action-specific cost function has the following form:

$$c(s_{ft}, a) = \sum_m (\gamma_g^{op} \cdot \mathbb{1}\{d_{ft}^m = 0\} + \gamma_g^{exp} d_{ft}^m a_m) + \gamma_g^{inv} a_{M+1}$$

where γ^{op} are the per-period operational costs (can be thought of as maintenance costs amortized over tracts), γ^{exp} are the per-kilometer costs of expansion which depend on the minimum build distance, and γ^{inv} are investment costs. Under this specification, the cost to move between each successive capacity tier is assumed to be constant. Each network technology group, $g \in \{cable, fiber, overbuilder\}$, has its own set of costs. By construction

[18] Innovation between generations—e.g. the shift from DOCSIS 2.0 to DOCSIS 3.0 for cable technologies involving both upgrades to both the physical hardware and algorithms for data transmission—is taken to be exogenous.

[19] Because revenues π are 'observed' in the dynamic model, this parameter η is identified. See Sweeting (2013) for a more thorough argument.

of the state space, the total cost can be decomposed into

$$c(s_{ft}, a) = \sum_m c_m(s_{ft}^m, a_m)$$
$$c_m(s_{ft}^m, a_m) = \begin{cases} \gamma_g \cdot \mathbb{1}\{d_{ft}^m = 0\} + \gamma_g^{exp} d_{ft}^m a_m & m \leq M \\ \gamma_g^{inv} a_m & m = M+1 \end{cases}$$

which will come in handy with estimation.

Let $\sigma_f : \mathcal{S}^{M+1} \rightarrow \mathcal{A}^{M+1}$ denote firm f's policy and $\sigma = \{\sigma_f\}_f$ denote the collective strategy profile of the market. The incumbent firm's ex-ante value function under this policy can be expressed as

$$V^\sigma(s_{ft}) = \mathbb{E}_\epsilon[\max_a\{u(s_{ft}, a) + \eta\epsilon_{ft}(a) + \beta\mathbb{E}_{\sigma_{-f}}[V^\sigma(s_{ft+1})|s_{ft}, a]\}]$$

where the transition to state s_{ft+1} depends on the firm's current state, action vector, and the actions taken by all other competitors in the market σ_{-f}. Meanwhile, the potential entrant of exogenously assigned technology type g trades off the regional entry cost against future profits:

$$V^e(s_{ft}) = \max\{0, -\gamma_g^{hub} + \beta\mathbb{E}[\max_a V^\sigma(s_{ft+1}(a))]\}$$

Future profitability depends on the initial set of tracts that the firm enters into, $\{m \mid a_m = 1\}$.

Let $Q^\sigma(s_{ft}, a)$ be firm f's action-specific value function:

$$Q^\sigma(s_{ft}, a) = u(s_{ft}, a) + \beta\mathbb{E}_{\sigma_{-f}}[V^\sigma(s_{ft+1})] \tag{1}$$
$$\implies V^\sigma(s_{ft}) = \mathbb{E}_\epsilon[\max_a\{Q^\sigma(s_{ft}, a) + \eta\epsilon_{ft}(a)\}]$$

Thus, given the Gumbel error assumption, the conditional choice probabilities (CCPs) under strategy profile σ can be written as

$$\mathbb{P}^{\sigma}(a|s_{ft}) = \frac{\exp(Q^{\sigma}(s_{ft}, a)/\eta)}{\sum_{a'} \exp(Q^{\sigma}(s_{ft}, a')/\eta)}$$

Next, I discuss further assumptions taken to pinpoint the strategy profile played by firms in the market.

3.4 Equilibrium

The equilibrium concept I employ is the restricted experience-based equilibrium (EBE) from Fershtman and Pakes (2012). Formally, the tuple $(\mathcal{R}, \mathcal{P}^*, W^*)$ is a restricted EBE if it satisfies the following conditions:

C1. $\mathcal{R} \subseteq \mathcal{S}$ is a recurrent class

C2. Strategies are optimal on $\mathcal{R}$: $\forall s \in \mathcal{R},\ \mathcal{P}^*(s) = \arg\max_a W^*(s, a)$

C3. The action-specific value function is consistent for all feasible strategies (not just σ^*) from states in $\mathcal{R}$:
$\forall s \in \mathcal{R},\ W^*(s, a) = r(s, a) + \beta \mathbb{E}[W^*(s', \mathcal{P}^*(s'))|s, a]$

This is similar to but weaker than a Markov Perfect Equilibrium (MPE) as it does not restrict agents' beliefs in states outside of the recurrent class (i.e. states that are not visited repeatedly and frequently for agents to build 'experience-based' beliefs). EBEs allow for asymmetric information (i.e. the unobserved plan qualities ξ—one of the key profit differentiators in the market that helps explain firm expansion behavior) at the cost of admitting more equilibria than an MPE. This is not an issue for estimation, as $\mathcal{P}^*$ is directly observed, but will need to be addressed with further assumptions in counterfactual analysis when the model primitives change.

The action-specific value function $Q^*(s_{ft}, a)$ satisfying (1) (along with optimal policy $\sigma^*(s_{ft}) = \arg\max_a Q^*(s_{ft}, a)$ and the class of all paths following σ^* from any initial $s_0 \in \mathcal{S}$)

constitute a restricted EBE in the dynamic broadband model. Conditions C1 and C2 are met by construction. Condition C3 is satisfied by reward function $r(s_{ft}, a) = \Pi(s_{ft}, a) + \beta(\eta\gamma - \log(2)/2)$ where γ is Euler's constant. The proof can be found in Appendix C.

Fershtman and Pakes (2012) provide a straightforward algorithm for estimating the conditional value function in EBEs using Q-learning. However, the high dimensionality of the continuous state space and combinatorial action space—for even just a single firm, let alone multiple players—make Q and σ intractable and the broadband problem very difficult to solve in practice. In the following section, I introduce a method to decompose the value functions while ensuring that optimality conditions are still maintained.

4 Estimation

Estimation of the model occurs in 2 stages. First, I estimate the static demand parameters and the firms' marginal costs. Then, I estimate the fixed, entry, and investment costs governing the firms' dynamic decisions. I assume that all observed states and actions in the data are on the equilibrium path (i.e. in the recurrent class) and therefore the EBE conditions hold.

4.1 Demand

The static primitives of interest are the mean preferences for price, speed, and other plan qualities $\theta_1 = (\beta_p, \beta_b, \beta)$ as well as the nonlinear household-specific parameters $\theta_2 = (\sigma_p, \sigma_b)$. Following the standard approach from Berry et al. (1995) and Conlon and Gortmaker (2020), I use generalized method of moments (GMM) to estimate θ_1, θ_2. The moment conditions of the GMM estimator are given by

$$g_D(\theta_1, \theta_2) = \mathbb{E}[\xi_{jt}(\theta) | Z_{jt}] = 0$$

Instruments Z_{jt} must account for the endogeneity of both prices and download speeds. On the price side, I leverage market-specific cost shifters such as population density and terrain ruggedness, which I assume are independently drawn characteristics that affect firm marginal costs and therefore plan prices. For download speeds, I use the lagged maximum supported network speeds residualized by cumulative investment (i.e. the initial tract capacity in period $t = 0$, which I assume varies independently of current period ξ_t). Under the assumption that infrastructure upgrades require substantial installation time compared to how rapidly plan speeds can be adjusted, residual speed capacities limit firm's offered plan speeds but are orthogonal to unobserved demand shocks. Finally, I also include the set of interactions between within-market demographic means and the aforementioned instruments. These instruments help identify the household heterogeneity parameters θ_2 under a linear IV approximation, see Conlon and Gortmaker (2020) for more details.

4.2 Marginal Costs

In order to estimate the firms' marginal costs c_0 and c_1, I assume that advertised plans (the market-specific prices and speeds reported in the URS data) satisfy the firm's profit-maximization FOCs. Let p^o and b^o denote these observed prices and speeds, respectively. I construct the following moments:

$$g_{MC}(c_0, c_1) = \mathbb{E}\begin{bmatrix} \Omega^p \cdot (p^o - c_0 - c_1 b^o) + s(p^o, b^o) \\ \Omega^b \cdot (p^o - c_0 - c_1 b^o) - c_1 s(p^o, b^o) \end{bmatrix} = 0$$

and use GMM to estimate the marginal cost parameters. Ω now denotes the block-diagonal matrix of elasticities stacked across all firms, markets, and periods. Likewise, p^o, b^o, c_0, and c_1 are also stacked vectors. Density plots of the estimated marginal costs can be found in Appendix D.

4.3 Supply

The dynamic parameters of interest are the technology-specific cost parameters $\theta = (\gamma^0_{Cable}, \gamma^0_{Fiber}, \gamma^{exp}_{Cable}, \gamma^{exp}_{Fiber}, \gamma^{exp}_{Over}, \gamma^{inv}_{Cable}, \gamma^{inv}_{Fiber})$ and the scale parameter on the cost shock, η. At a high level, estimation of these parameters proceeds in two stages. In the first stage, I recover estimates of the firms' value functions and optimal policies for a fixed parameter vector θ; in the second stage, I maximize the CCPs implied by these estimates—evaluated at the observed states and actions in the data—over potential θ and η.

Fershtman and Pakes (2012) show how a standard Q-learning algorithm can be used to estimate Q^* and σ^*; i.e.:

$$Q^{k+1}(s_t, a_t) = (1 - \alpha(k))Q^k(s_t, a_t) + \alpha(k)(r(s, a) + \beta \max_{a_{t+1}} Q^k(s_{t+1}, a_{t+1})) \tag{2}$$

converges to Q^* as iterations $k \to \infty$ for appropriate values of learning rate $\alpha(k)$[20]. Note that the transition to s_{t+1} depends on competitors' actions; I assume that firms have rational expectations. However, the state space of the broadband model spans $\times_{f=1}^{F} \mathcal{S}_f$ where F is the number of firms (incumbent and entrant) and $S_f \in \times_M(\mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{Z}^+) \times \mathbb{R}^4$. The action space spans $\{0, 1\}^{M+1}$ where M is the number of tracts in the broader market. Both make it computationally intractable to apply (2) to the broadband game. Naturally, there are ways of reducing the state and action spaces—for example, one could keep track only of the markets on the boundary of the firm's network or define clusters of tracts over which the firm makes the same decision. The first case, unfortunately, still runs into tractability issues as the firm's network grows (in the *smallest* potential state in my data, when Verizon operates in just 4 tracts, it can still potentially expand into 17 adjacent markets). The second case faces its own implementation challenges: how big should clusters be (if they're too small, computational issues remain; if they're too big, uniformity of actions may be violated in the data); should the clusters be the same for all firms (if yes, they will be

[20] Fershtman and Pakes (2012) use $\alpha(k) = 1/h^k(s_t)$ where h^k is the number of times state s_t has been visited in iteration k.

difficult to define as there is little consistency in behavior across firms; if no, it's unclear how clusters will be defined for potential entrants). These details make results very sensitive to the specific implementation. Another (extreme) simplification is to consider the firm's problem as a collection of independent tract-level optimization games, reducing the state space to $(\mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{Z}^+) \times \mathbb{R}^4$ and action space to $\{0,1\}^2$. However, tracts are necessarily interdependent due to the network structure of broadband infrastructure, so these local optima do not necessarily aggregate to the network-level optimal policy. Thus, rather than modifying the game, I instead opt to modify the estimation algorithm. I propose a novel reinforcement learning algorithm that decomposes the conditional value function into functions of tract-specific state-action pairs while accounting for strategic incentives arising from network interactions.

4.3.1 Algorithm

First, consider the decomposition of the action space. Let $Q_a(s, a^m)$ and $r_a(s, a^m)$[21] be tract m's contributions to the conditional value function and flow profits, respectively, that satisfy

$$Q(s, a) = \sum_m Q_a(s, a^m)$$

$$r(s, a) = \sum_m r_a(s, a^m)$$

where a^m is the m-th component of the action vector a. In the cooperative reinforcement learning literature, Kok and Vlassis (2004) show that the update rule

$$Q_a(s, a^m) \leftarrow (1 - \alpha) Q_a(s, a^m) + \alpha (r_a(s, a^m) + \beta \frac{1}{M+1} \sum_{m'} \max_{a^{m'}} Q_a(s', a^{m'})) \tag{3}$$

[21] Time and firm subscripts are omitted for notational clarity, but everything is defined within a single firm-period.

converges to a policy Q^*_a satisfying $\sum Q^*_a = Q^*$[22]. Intuitively, when rewards are pooled, cooperation across markets is incentivized. For ease of notation, the $M+1$th 'tract' corresponds to the upgrade decision and always yields weakly negative single period payoffs due to zero profits. Network interactions arise when actions in adjacent $m' \neq m$ affect the state transition of s^m. Summing this expression over all agents gives the standard Q-learning update rule (2) for the global value function, which converges to Q^* in the limit when each (s, a) pair is visited infinitely often and the sequence α satisfies certain properties[23] Watkins and Dayan (1992).

Next, consider the decomposition of the state space. Define $Q_s(s^m, a^m)$ and $r_m(s^m, a^m)$ that satisfy

$$Q_a(s, a^m) = \sum_{m'} Q_s(s^{m'}, a^m)$$
$$r_a(s, a^m) = \sum_{m'} r_s(s^{m'}, a^m)$$

In the modular reinforcement learning literature, Russell and Zimdars (2003) and Sprague and Ballard (2003) both show that the update rule

$$Q_s(s^m, a^m) \leftarrow (1-\alpha)Q_s(s^m, a^m) + \alpha(r_m(s^m, a^m) + \beta Q_s(s^{m'}, a^{m'})) \tag{4}$$

converges to Q^*_s satisfying $\sum Q^*_s = Q^*_a$ in the limit that (s^m, a^m) are visited infinitely often. Importantly, $a^{m'}$ is the successor action chosen according to the aggregate policy, $a^{m'} = \arg\max_{a^m} Q_a(s, a^m)$ and not necessarily the locally optimal action $\arg\max_{a^m} Q_s(s^m, a^m)$. The intuition is that the aggregate policy in the future state serves as a coordination device so that deviation in the current state (from the globally optimal to the locally optimal) is

[22]There are infinitely many ways of partitioning Q beyond equal shares $\frac{1}{M+1}$. One necessary condition on the partition function h is that $O(Q) = O(\sum h(Q))$, otherwise the value function explodes in magnitude and the update rule will not converge. In my estimation, I use a modified update of $\frac{1}{M} \sum_{m' \neq M+1} \max Q_a(s', a^{m'})\ \forall m \neq M+1$ and $\sum_{m'} \max Q_a(s', a^{m'})$ when $m = M+1$.

[23]Specifically, $\sum_t \alpha(t) = \infty$ but $\sum_t \alpha^2(t) < \infty$. One such sequence that satisfies this property is $\alpha = 1/t^\omega$ for $\omega \in (0.5, 1]$. In my estimation, I use $\omega = 0.9$.

disincentivized. Network interactions affect both the transition of the state from s to s' as well as the continuation value Q_s through $a^{m'}$.

Combining these two ideas, I propose an update rule that depends only on tract-specific state-action pairs to converge to the desired Q^*_m equilibrium value functions:

$$Q_m(s_t^m, a_t^m) \leftarrow (1-\alpha)Q_m(s_t^m, a_t^m) + \alpha(r(s_t^m, a_t^m) + \beta \frac{1}{M+1} \sum_{m'} Q_m(s_{t+1}^{m'}, a_{t+1}^{m'})) \quad (5)$$

where a_{t+1} is chosen according to the firm's greedy policy σ.

Finally, with the conditional value function estimates $\hat{Q}_m$, I use maximum likelihood to recover the dynamic parameters of interest:

$$\begin{aligned}
\hat{\theta}, \hat{\eta} &= \arg\max_{\theta,\eta} \hat{\mathbb{P}}(a^o | s^o; \theta, \eta) \\
&= \arg\max_{\theta,\eta} \Pi_{f,t,m} \frac{\exp(\hat{Q}(s_{ft}^{o,m}, a_{ft}^{o,m}; \theta)/\eta)}{\exp(\hat{Q}(s_{ft}^{o,m}, 0; \theta)/\eta) + \exp(\hat{Q}(s_{ft}^{o,m}, 1; \theta)/\eta)} \\
&= \arg\min_{\theta,\eta} \sum_{f,t,m} -\ln(\frac{\exp(\hat{Q}(s_{ft}^{o,m}, a_{ft}^{o,m}; \theta)/\eta)}{\exp(\hat{Q}(s_{ft}^{o,m}, 0; \theta)/\eta) + \exp(\hat{Q}(s_{ft}^{o,m}, 1; \theta)/\eta)}) \quad (6)
\end{aligned}$$

where s^o and a^o denote the states and actions observed in the data, respectively. In practice, because the likelihood function is discontinuous and non-differentiable[24], I use grid search to solve the optimization. The steps of the full estimation procedure can be found in Appendix E.2; discussion of how I modify the update rule to accommodate continuous state variables can also be found in Appendix E.1.

4.3.2 Illustrative Example

To highlight the key strategic incentives and demonstrate how the proposed algorithm accounts for these mechanisms, I propose the following toy example of a single firm in an artificially constructed 5-tract market as illustrated in Figure 5. In the diagram, nodes correspond to tracts, r_m denotes the potential profits earned in tract m each period, and edges

[24] The summation in the update function depends on M threshold rules for M binary outcomes.

indicate adjacency with d_{mn} capturing the minimum build distance between tracts m and n (not drawn to scale). A node is filled if the firm has entered the tract.

As in the full game, the firm makes entry decisions in every adjacent market and there is no limit to the number of markets the firm expands into in each period. Note that within each period the expansion path is not explicitly modeled; instead, the firm is assumed to build along the shortest possible route to connect a new tract. Each period, the firm also makes a single investment decision—should the firm choose to invest, r_m increases deterministically by 1 in every active tract (up to a profit cap of 5).

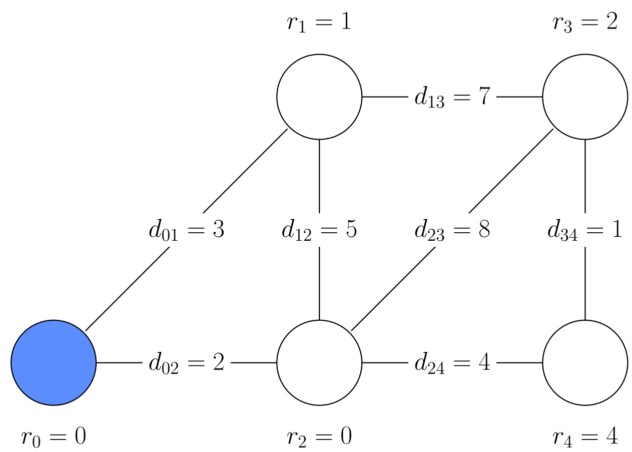


Figure 5: Network example with limited entry. The firm is active in the shaded nodes; r_i denotes the reward in market i; d measures build distances between markets (not drawn to scale).

Suppose, to simplify the decision-making process, the firm has delegated the expansion and upgrade decisions to managers in each tract who seek to maximize their own profits. Managers have access to the full state s, but, even with full information, face a coordination problem as they have no incentive to consider the impact of their action on the full network. To see this, consider the decision of the manager in market 2. It is clear that, in isolation, it is never profitable to enter this tract. However, entry into tract 2 opens up the possibility of expansion into the profitable tract 4 in a future period. Thus, from the perspective of the firm, it may be optimal for one manager to take a loss in order for another to make greater profits. Similarly, the manager in charge of upgrade decisions never finds it optimal to upgrade as all benefits are reaped by other managers. Profit sharing as in (3), where each manager receives a portion of the firm's total profits, can incentivize such coordination while decoupling the decisions in each location.

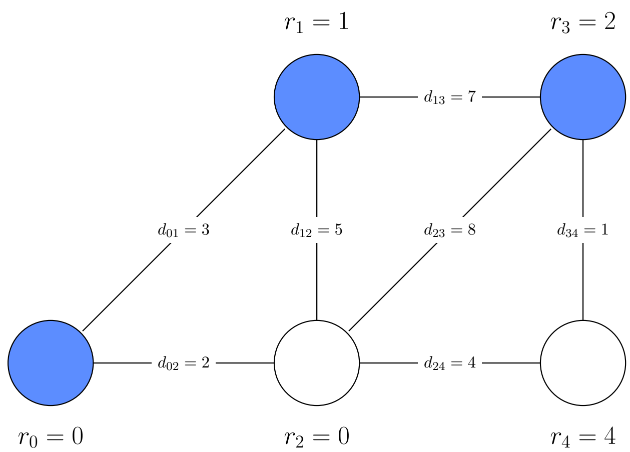


Figure 6: Network example with additional expansion. The firm is active in the shaded nodes; r_i denotes the reward in market i; d measures build distances between markets (not drawn to scale).

There remains, however, the issue of the large state space—realistically, managers are not making decisions based on the full state. Suppose, instead, that managers' information access is limited to just the state of their tract[25]. A new coordination problem arises as managers now cannot observe the impact of their action on the full network. To illustrate this, suppose the firm's network were instead given by the diagram in Figure 6. Given tract 3's proximity to tract 4, it is inefficient and unprofitable for the (unaware) manager in tract 2 to enter. In other words, the continuation value of entering a tract, which is a function of the profitability of unentered tracts, depends on the scope of the rest of the network. Consider, instead, a modified policy function (4) where managers propose locally optimal actions based on limited information, the firm picks from these proposals to maximize its estimate of the global value function, and managers execute the firm's designated actions. This significantly reduces the computational complexity because, rather than learning the

[25]More generally, suppose there is some abstraction function mapping the full state space to a lower-dimensional state for each tract: $f_m : \mathcal{S} \to \mathcal{S}_m$.

β	Coefficient	Interaction with Demographics			
		Married	Kids	Income	Income^2
Price	-9.526***	1.905***	-0.514	3.386***	0.072
	(1.077)	(0.667)	(0.470)	(1.158)	(0.321)
Download	1.566***	0.083	-0.110	-0.797***	-0.014
	(0.202)	(0.132)	(0.096)	(0.221)	(0.063)
Upload	0.728***				
	(0.045)				
Capped	-0.534***				
	(0.041)				

Table 6: Estimates from demand model specification with technology, period, firm, and city FEs; limited to plans with market share $\geq 0.1\%$ in all three metropolitan areas.

optimal policy over the full state space, the firm only needs to evaluate it at a select number of points.

Appendix E.3 provides additional details on how I verify the performance of the proposed algorithm in this artificial setting.

5 Results

5.1 Demand

Parameter estimates for the full model specification using data from all three metropolitan areas are shown in Table 14. In the dynamic game, upload speeds are omitted for ease of computation, but this is not entirely unrealistic to how households select internet plans[26] and estimates do not change significantly. Estimation results for this specification and for each metropolitan area separately can be found in Tables 6 in the Appendix; results are consistent across all models.

Results are intuitive: consumers prefer lower prices, faster speeds (upload and download), and no data usage caps. These estimates suggest that households value a \$16.44 decrease in monthly price—or a 22% discount on the average monthly plan price of \$76.44— as much

[26]From Goetz (2019): "92.5% of respondents in the 2013 Current Population Survey Internet Supplement list price, download speed, or reliability as the most important feature of service, from a list of choices that also includes upload speed, data usage caps, mobility, and bundling options."

as a 1% increase in download speeds. I also find that wealthier households are less sensitive to price (and also place less value on speeds). The same is true of married households, but the coefficients are not statistically significant. On the other hand, households with children are more price sensitive and have greater preference for faster speeds, plausibly due to more devices in the residence and thus higher bandwidth requirements. Households are quite elastic—the median own-price elasticity is -1.95—however, there is substantial variation across markets and time, as shown in Figure 13. The median own-(download) speed elasticity is 2.09, which suggests that consumers are slightly more sensitive to speed improvements than to price changes.

Table 17 shows the price diversion ratios at the firm level. I find that the majority (69-72%) of Comcast customers switch to the outside option (no internet) following a price increase—plausibly due to the fact that Comcast, the incumbent cable provider, is the only high-speed option in the market or, even, the only option households are aware of—the remainder are fairly evenly split between RCN (primarily cable) and Verizon (primarily fiber). Along this same vein, customers of the cable and fiber entrant firms (RCN and Verizon) are much more likely to switch to Comcast (69-72% and 64-80%, respectively) than the low-speed satellite providers (HNS and Viasat) or the outside option. HNS and Viasat customers primarily opt for the option of no internet (58% and 47%, respectively); a smaller fraction leave for Comcast (31% and 39%, respectively) compared to customers of RCN and Verizon. The download speed diversion ratios in Tables 19 display similar trends. This suggests that households who choose satellite internet have stronger preferences for lower prices and may benefit from increased entry if competition drives down prices; Comcast customers, on the other hand, show stronger preferences for speed and could benefit from greater availability of other high-speed competitors.

Technology-level price and download speed diversion ratios can be found in Tables 18 and 20. Results suggest that cable customers largely switch to the outside option in response to a price increase or speed decrease—likely due to lack of viable alternative choices. Of the remaining customers, the vast majority opt for fiber and the rest choose DSL or satellite. Fiber customers display similar trends—one-third switch to the outside option and two-thirds choose cable.

5.2 Supply

Table 7 presents the estimation results for the dynamic cost and scale parameters governing firms' expansion and investment decisions. The estimates suggest that operational costs are fairly small, at around \$250 and \$2000 per tract per year for cable and fiber, respectively. One plausible explanation for the lower operational costs faced by fiber providers is that the firms' networks require less maintenance and upkeep because they are newer. On the other hand, the per-kilometer costs of expansion are fairly comparable for cable and fiber, at approximately \$61.8k[27] and \$56.3k (equivalently, \$99.5k and \$90.5k per mile), respectively. Given that installation, which does not differ significantly between the two technologies, contributes the vast majority of expansion costs, this makes intuitive sense. Moreover, these results fall within industry benchmarks for fiber, which report that a mile of fiber optic cable costs between \$77k and \$123k for suburban to urban areas depending on factors such as the method of installation, the density of the fiber, and the type of terrain[28]. Finally, I find that it costs just around \$4.7 million to upgrade cable hardware; fiber is significantly cheaper at \$1.3 million.

5.2.1 Model Fit

Next, I evaluate the model's ability to predict a number of (untargeted) market characteristics. Simulated outcomes average across 100 paths drawn according to the policy and probabilities derived from the value function estimates. Figure 7 compares the expansion be-

[27] The only cable expansion I observe is done by the overbuilder, RCN, which pays a discounted expansion fee that I assume is additive and constant across technologies for identification). The baseline per-kilometer cost for regular cable providers is the difference between these two estimates.

[28] Source: https://www.fierce-network.com/broadband/underground-fiber-drives-deployment-costs

		Coefficient	95% CI
γ_0			
	Cable	0.499	[0.485, 0.546]
	Fiber	0.147	[0.111, 0.163]
γ^{exp}			
	Cable	6.183	[6.069, 6.187]
	Fiber	5.626	[5.607, 5.637]
	Overbuilder	-0.854	[-0.872, -0.842]
γ^{inv}			
	Cable	47.057	[46.91, 47.067]
	Fiber	13.679	[13.674, 13.754]
η^{exp}		5.713	[5.711, 5.736]
η^{inv}		46.244	[46.238, 46.29]

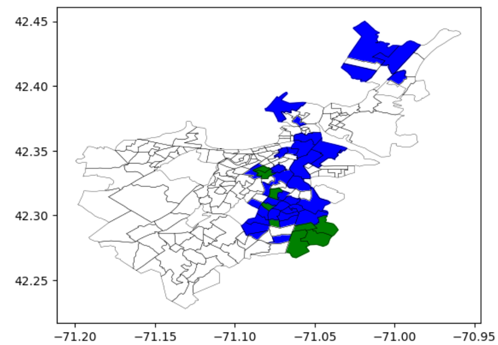
Table 7: Boston cost estimates in $10,000 (except investment, which is reported in $100,000).

havior observed in the data (left column) against the modal[29] simulated path (right column) of Verizon fiber expansion. I find that the model fits the general direction of expansion—first south, then west—fairly well, although the precise timing of entry in individual tracts may lag by a period or two. Table 8 describes tract expansion statistics over time for Verizon fiber; the model does well in matching the volume of expansion by Verizon every period[30]. The model also matches the total activity of RCN but predicts more gradual expansion than that observed in the data. This is because RCN exits a few markets in the second half of the data; however, the model restricts firms from exiting, so it predicts more moderate growth that slowly catches up to the level of observed activity at the end of 2019. I also find that the model matches the investment behavior of all firms (see Table 9), although confidence intervals are quite wide due to the limited number of observations in the data.

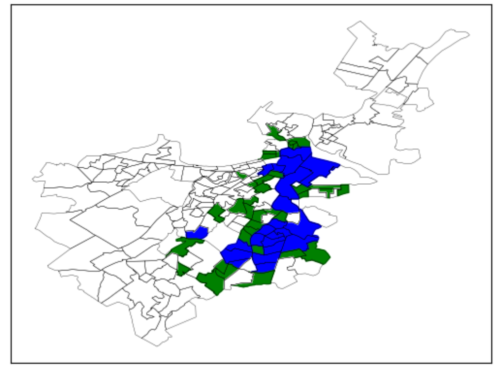
Figures 8 and 9 plot plan prices and download speeds, respectively, over time averaged across tracts and simulations. The model does well in predicting the speeds and prices offered by all of the cable and fiber providers except RCN cable. I consistently under-predict

[29] I assume a tract has been entered if the firm is active in it in the majority of simulations.

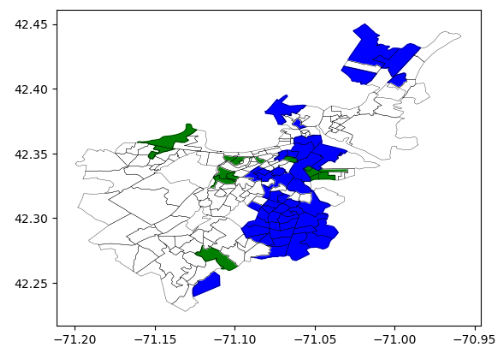
[30] There is a slowdown between 2016 and 2017 that can be attributed to a strike in May 2016 that involved 40,000 workers and spanned 45 days. See https://www.nytimes.com/2016/05/31/business/verizon-reaches-tentative-deal-with-unions-to-end-strike.html for more details. This is not in my model, as such predicted expansion exceeds observed entry during this period.



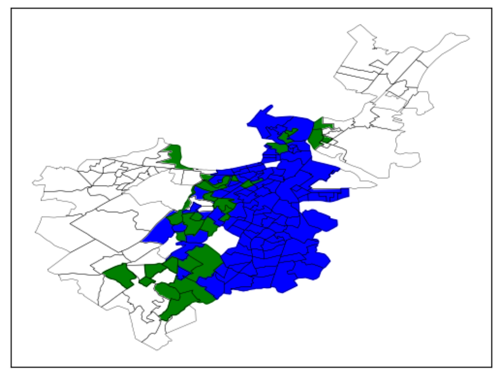
(a) 2017Q2



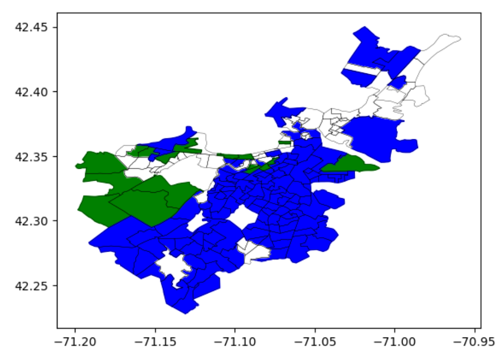
(b) 2017Q2



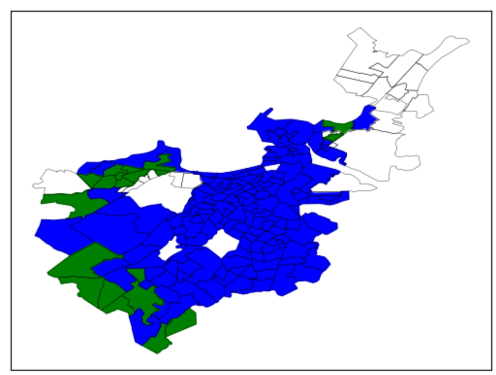
(c) 2018Q2



(d) 2018Q2



(e) 2019Q2



(f) 2019Q2

Figure 7: Observed (left) vs predicted (right) expansion of Verizon fiber across Boston census tracts. Blue indicates services offered in previous period; green indicates new expansion.

both the prices and download speeds for this firm. The lower-than-predicted optimal prices could be due to promotional pricing targeted towards specific households in order to capture market share from the established cable incumbent, Comcast. These promotions are not observed in the data but could help motivate expansion into seemingly unprofitable areas.

Period	Sim. Active	Std. Error	Obs. Active
2015Q2	4.00	0.00	4
2015Q4	12.72	2.39	6
2016Q2	24.68	3.53	20
2016Q4	40.52	6.17	34
2017Q2	59.96	7.45	43
2017Q4	76.32	8.90	47
2018Q2	93.04	10.06	62
2018Q4	110.84	8.97	121
2019Q2	125.40	9.14	137
2019Q4	139.28	7.99	155

Table 8: Mean predicted versus observed number of active tracts per period for Verizon fiber.

Firm	Action	Sim. Count	Std. Error	Obs. Count
RCN Cable	invest	1.95	0.92	3
RCN Cable	stay	6.36	1.11	5
RCN Fiber	invest	2.21	1.06	2
RCN Fiber	stay	5.88	1.13	6
Verizon Fiber	invest	2.32	0.78	1
Verizon Fiber	stay	5.96	1.06	7
Comcast Cable	invest	2.46	0.98	2
Comcast Cable	stay	5.64	1.08	6

Table 9: Mean predicted versus observed investment actions by provider.

This same behavior is not true of fiber because the fiber incumbent is not well established (undergoing rapid expansion during this period). The discrepancy in speeds likely arises due to restrictions in the stage game for computational tractability [31]. Given that these are endogenous outcomes in an extremely high-dimensional dynamic game, the model does a decent job at matching the firms' responses to different strategic incentives.

6 Counterfactuals

Broadband is critical infrastructure, and policymakers consider access and equity to be high priorities. Policies that seek to promote greater entry in the market must balance competitive gains against expansion and innovation incentives, making long-run welfare effects ambigu-

[31] I limit the set of possible download speeds to a finite set of the most commonly observed tiers and allow firms to offer at most 3 plans. Also, all else equal, the FOC of the speed equation necessarily results in a lower optimal speed if optimal prices are lower.

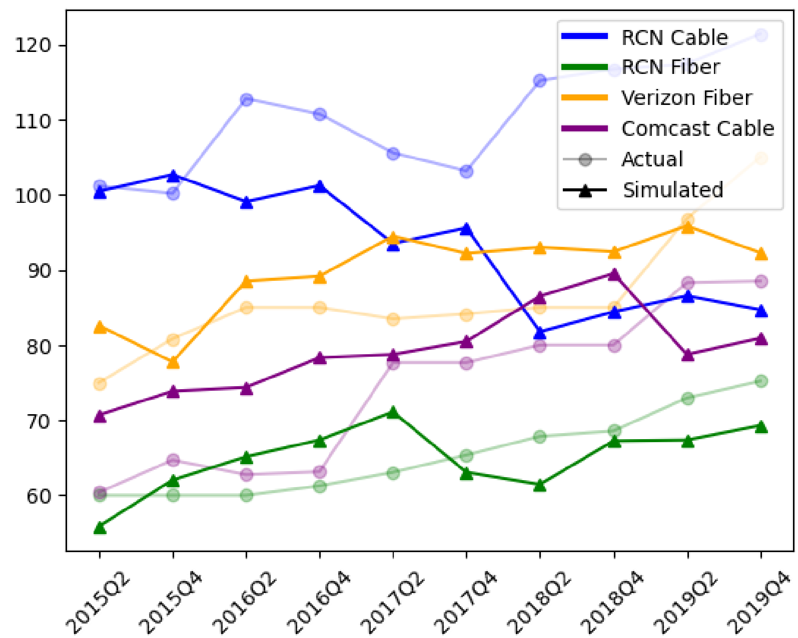


Figure 8: Mean predicted plan prices over time; observed prices shown in corresponding color for each firm.

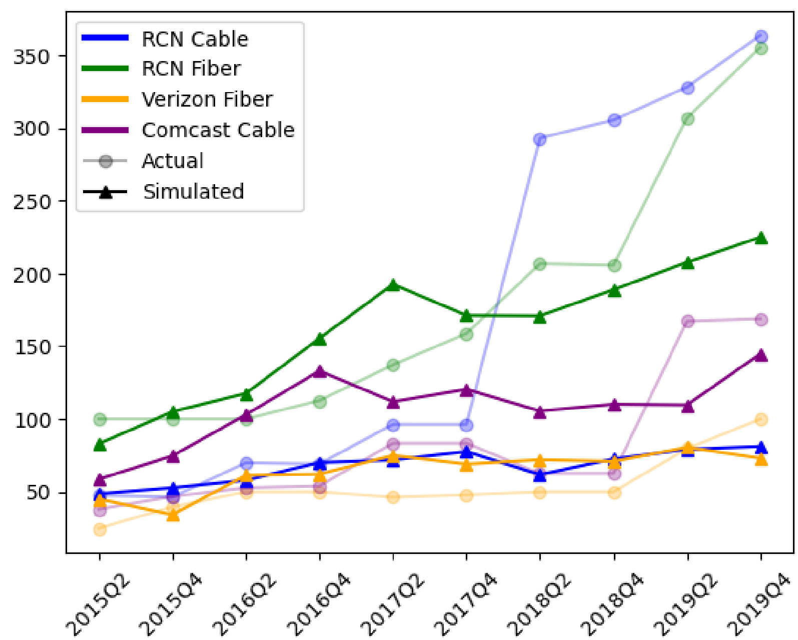


Figure 9: Mean predicted plan download speeds over time; observed speeds shown in corresponding color for each firm.

ous. I use the estimated model to shed light on some of the proposed policies. Outcomes of interest include the impact on static—affordability and plan quality—and dynamic—fiber availability and network quality, with a particular focus on cable capacity—competition. The structural framework endogenizes all of the above and allows me to speak to counterfactual welfare implications. The evaluated policy environments include:

Same-Technology Competition Ban. In this setting, I simulate a world where RCN, the regional cable and fiber competitor, is barred from entering the Boston market. This allows me to quantify—directly from the model and data without additional assumptions—the impact of entry competition on internet affordability and quality, fiber expansion, cable investment, and overall consumer welfare.

Municipal Broadband. I consider the possibility of municipal fiber entry in Boston in place of Verizon. The city can be thought of as a social planner (with the same costs and initial infrastructure as Verizon in 2015, the first period in my data) that sets speeds and prices according to the same profit-maximization objective in the static game, but makes dynamic decisions with the goal of maximizing the total social welfare from fiber. This assumes that the city does not internalize its impact on private cable providers and their customers. One of the benefits of municipal broadband is that it does not require national legislation or coordination to enact. In the post-2020 era, in response to rapidly growing internet demand, there has been a surge in municipal broadband interest.[32]

Local Loop Unbundling. In this counterfactual policy environment, all incumbents (cable and fiber) must lease their network to new entrants. The entrants have direct access to physical lines, which grants freedom to set their own plan speeds subject to network capacity constraints. Each firm must use their own equipment for data transmission along the network (i.e. they face different marginal costs). Only the incumbent can make expansion and investment decisions pertaining to their infrastructure. As compensation, entrants pay the

[32] According to a 2022 report by the Urban Institute.

incumbent a one-time connection fee for each line accessed[33] as well as a per-period leasing fee, which may or may not depend on total bandwidth usage. For ease of computation, I assume that potential entrants are randomly assigned a technology type to lease and that the fees paid to incumbents are the same across cable and fiber. I evaluate different combinations of the two fees to isolate the policy's effect on static competition, consumer outcomes, and incumbent profits.

6.1 Computation

In order to evaluate the equilibrium impact of each counterfactual, I solve for the updated value functions and optimal policies of each firm. The approach is similar to the estimation procedure with one caveat: I no longer observe firms' globally optimal policies (i.e. those that evaluate joint expansion decisions)[34]. To overcome this challenge, I nest the reinforcement learning algorithm within a path simulation cycle. At a high level, the procedure is:

1. Initialize Q^0 for all states in the data and all actions
2. Forward simulate from initial period states T steps according to the CCPs implied by the value function
3. Update Q^{k+1} at all states, treating the simulated path as observations of the globally optimal policy
4. Repeat 2-3 for $k = 1, ..., K$ iterations
5. Initialize a new Q^0 for all states in final simulated path. Repeat 2-4.

[33]On the upper limit, a cable line can serve roughly 500 households while a fiber line can only accommodate 128 (according to reports by Broadband Success Partners and Cisco, respectively). I assume access in each tract is an all-or-nothing decision; that is, in order to begin servicing a tract, an entrant must pay the connection fees for every line in that tract.

[34]This complicates the state-decomposition step of the algorithm, which relies on this 'signal' to ensure cooperation with limited information. To address this, I set initial conditions and parameters of the algorithm to encourage more exploration and run millions of update iterations. With enough exploration, the algorithm should be able to identify the global optimum should it differ from the local optima. Naturally, this is a very time consuming process; estimation converges on the order of seconds, counterfactual computation can take upwards of 20 hours. More details can be found in Appendix E.2.

6. Repeat 5 for N iterations. In the final run, iterate until Q converges.

The intuition is that with every 'data reset', the algorithm is getting closer to the true value function and, simultaneously, true actions that would be observed in the data under the counterfactual equilibrium policy. I reset after N iterations rather than storing all visited states in order to limit the computational complexity. Additional details on the solution method can be found in Appendix F.

With the value function estimates, I again simulate 100 action paths and average across iterations to evaluate firms' counterfactual strategies in equilibrium.

6.2 Results

In this section, I compare counterfactual outcomes against model fit simulations to better isolate changes due to competitive mechanisms rather than simulation error.

No Same-Technology Competition. Table 10 reports aggregate expansion and investment outcomes with and without competition from RCN. I find that, in the absence of another fiber provider, Verizon expands slower, prioritizing larger markets[35], and invests more heavily in infrastructure capacity[36]. The former is consistent with early mover advantages—Verizon has incentive to enter sooner when there exists another fiber provider in order to deter their entry; the latter is plausibly motivated by uncertainty in future profitability[37]. In contrast, Comcast cable, the largest provider in the market by a wide margin, cuts investment in capacity by 10%[38].

Figures 10 and 11 show price and download speed responses, respectively, over time.

[35] On average, Verizon operates in 11% fewer tracts each period, which translates to 10,700 fewer households with access to fiber (or 10.7% of all households in the broader market). Table 21 in Appendix A provides a more detailed view of fiber expansion outcomes over time.

[36] On average, the number of upgrade actions is roughly doubled.

[37] Greater competition leads to greater uncertainty about future profitability; the firm may be less willing to pay the high investment costs to establish itself in such a market.

[38] This is because without RCN, a high-speed entrant, Comcast faces significantly lower competitive pressure on speeds and therefore has less incentive upgrade capacity.

	Baseline	Tech. Ban	% Delta
Fiber Access			
Tracts	63.3	56.3	-11.0%
Households	99.7	89.0	-10.7%
Investment			
Fiber	112.9	117.3	+3.9%
Cable	123.6	111.0	-10.2%
Avg. Speeds	119.6	90.7	-24.2%
Avg. Price/mb	20.4	21.4	+4.9%
Producer Surplus			
Total	19728.7	14292.4	-27.6%
Incumbents	11981.3	14292.4	+19.3%
Consumer Surplus	51872.8	47809.7	-7.8%
Total Surplus	71601.4	62102.1	-13.3%

Table 10: Summary of market outcomes with and without same-technology competition.

Price is reported in dollars per log megabit to control for simultaneous changes in speeds while remaining consistent with the (log) units of consumer demand. I find that increased within-technology competition contributed to a modest improvement in the quality of plans offered by the cable incumbent but little to no impact on price, or on the static response by the fiber incumbent[39]. Table 10 also reports the expected discounted consumer surplus, producer surplus, and total surplus effects in this counterfactual environment. Consistent with economic intuition, consumers are worse off, incumbents benefit substantially, and total welfare declines in the absence of same-technology competition[40]. This suggests that any business stealing effects are significantly outweighed by the benefit to consumers when RCN is allowed to enter the market.

Finally, I use the demand model to break down the distributional consequences by household size and income. Figure 12 plots the percent change in consumer surplus by technology type for each demographic group. I find that low-income households, particularly those that

[39]On average across the entire market, consumers pay 5% more per megabit when RCN does not enter. The average download speed of a cable plan increases by 12 Mbps or 10%; the average speed of a fiber plan increases by 4 Mbps or 4%. However, this is driven primarily by a change in the composition of firms and variety of plans rather than competitive responses by the incumbents.

[40]Profits increase 19% collectively for the two incumbents, consumer surplus declines by 8%, and total welfare in the market drops by 13%.

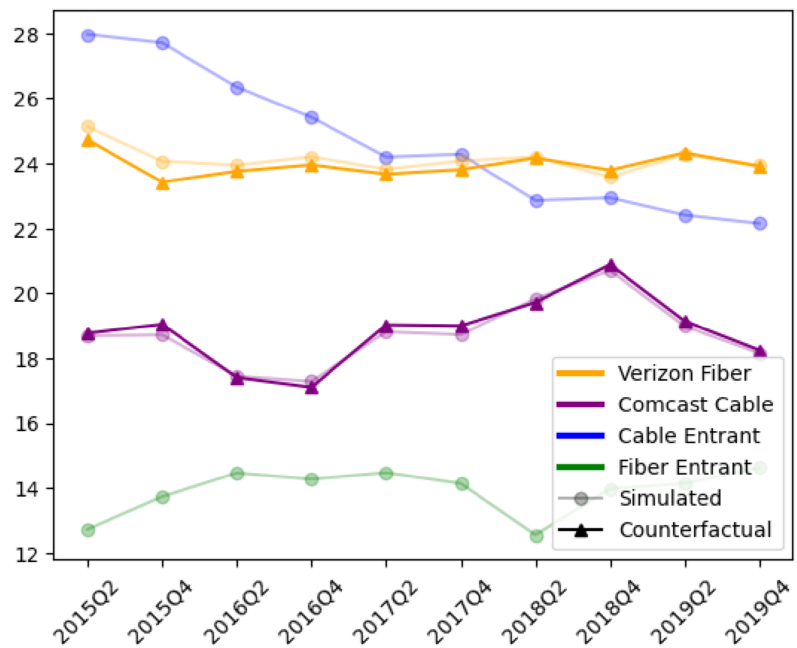


Figure 10: Plan price per megabit over time; darker lines correspond to counterfactual outcomes under the entry ban while lighter lines correspond to model simulations.

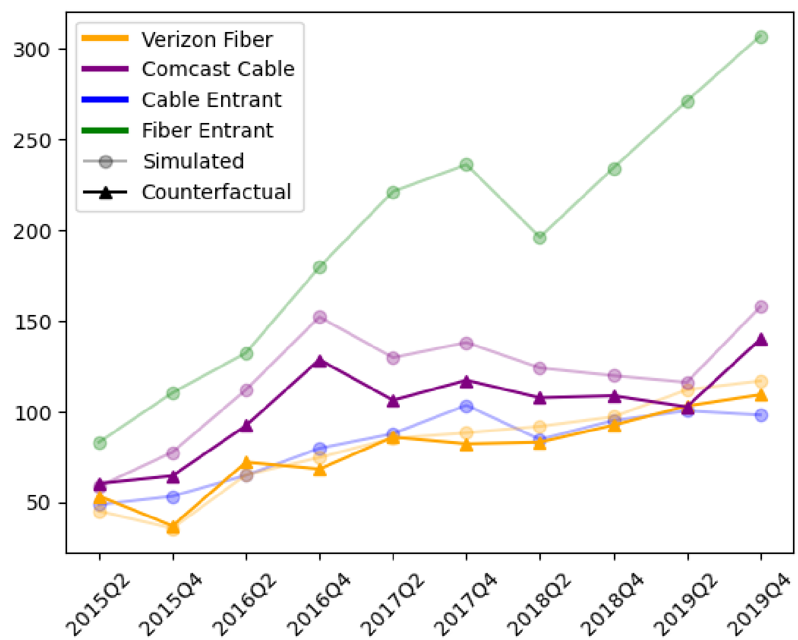


Figure 11: Plan download speed over time; darker lines correspond to counterfactual outcomes under the entry ban while lighter lines correspond to model simulations.

are married, gain the most from the entry of a new fiber provider[41]. Medium and high

[41]These households are more price sensitive but simultaneously have greater taste for speed, which makes

income households are more insulated from the changes in competitive outcomes. On the other hand, the absence of a second cable competitor hurts all income groups substantially and equally, with the exception of unmarried, high income households[42]. Figures 14 and 15 further decompose the impact on consumer surplus by static versus dynamic outcomes, respectively[43]. The results indicate that benefit to consumers from same-technology entry in broadband is dominated by dynamic market outcomes (i.e. access and variety) rather than static channels (i.e. quality and affordability). This is particularly important for low-income households, where the magnitude of the dynamic impact is nearly 10x that of static outcomes.

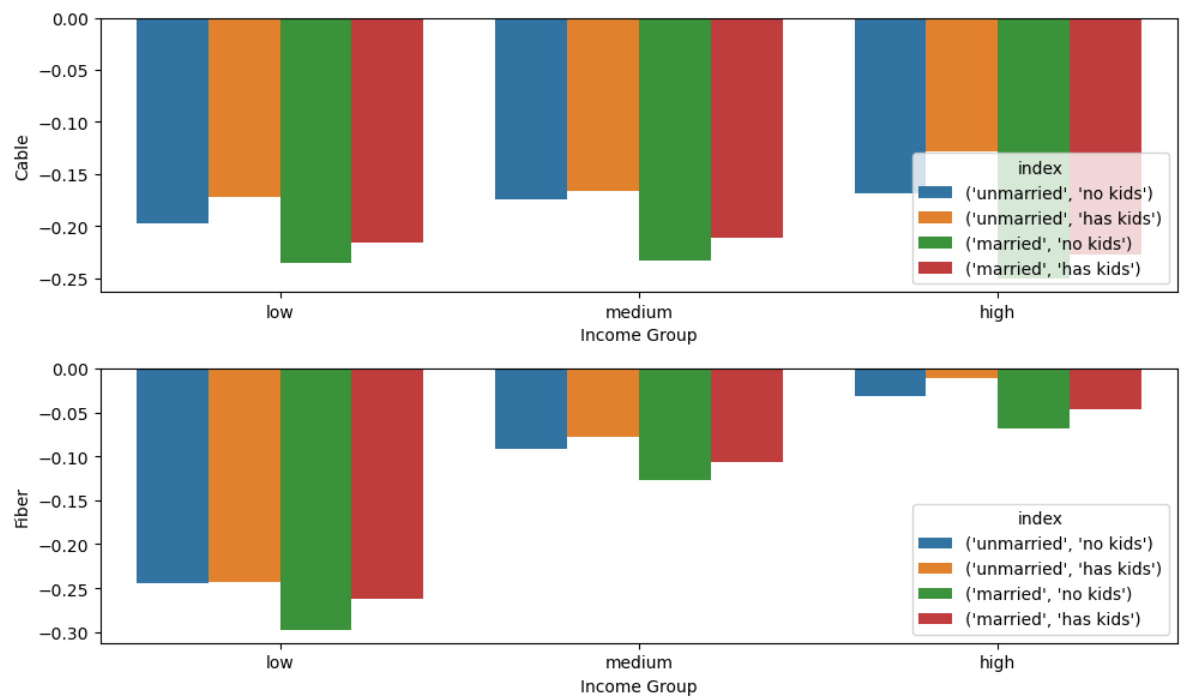


Figure 12: Change in consumer surplus without same-technology entry by demographic group and access technology.

Municipal Broadband. Table 11 shows the market's dynamic response to a municipal fiber provider that acts as a social planner. I find that the socially optimal degree of fiber

them particularly suited for RCN fiber.

[42]This demographic is the most inelastic across all groups.

[43]The change in consumer surplus due to strictly dynamic outcomes is computed by assuming firms are not allowed to update their menu of plan prices and speeds; the static delta is the difference between this dynamic intermediate and the full counterfactual.

	Baseline	Planner	% Delta
Access			
Tracts	63.3	68.1	+7.5%
Households	99.7	107.7	+8.0%
Investment			
Fiber	112.9	98.8	-12.5%
Cable	123.6	112.6	-8.9%
Avg. Speeds	119.6	84.7	-29.1%
Avg. Price/mb	20.4	22.5	+10.6%
Producer Surplus	17473.1	18178.4	+4.0%
Consumer Surplus			
Total	50806.7	50551.2	-0.5%
Fiber Only	27977.9	28394.9	+1.5%
Total Surplus	68279.8	68729.6	+0.6%

Table 11: Summary of market outcomes with and without a municipal fiber provider.

access is greater than the availability under private provision[44]. Compared to a private provider, the municipal network is also slightly more likely to expand into smaller markets (by population). However, investment in infrastructure capacity declines across the board for public and private providers alike[45]. These changes translate into static outcomes from the stage game—average plan speeds fall and prices increase[46]. It appears that the social planner trades off increased access for a subset of households against lower quality and more expensive plans on average for the entire market. Time series trends in prices and speeds can be found in Figures 16 and 17 in Appendix A.

Interestingly, welfare analysis suggests that producer profits increase modestly by 4%, but total consumer surplus *decreases* due to the social planner. This is because the municipal provider is only concerned with the market outcomes from fiber; it does not internalize the competitive effects on cable providers and their customers, who face lower quality and higher priced plans. When welfare is isolated to just the contribution from fiber in the simulations,

[44]The municipal network expands into 5 (or 8%) more markets on average each period. Equivalently, an additional 8,000 household have access to fiber under the municipality.

[45]The municipality invests 13% less because it does not fully internalize the returns to higher capacity/speeds; private cable providers invest 9% less on average due to decreased speed competition.

[46]Offered speeds decrease by 6%, 11%, and 7% for the municipal fiber provider, cable incumbent, and cable entrant, respectively. Price changes for these firms are nominal, averaging 0.5%, and are mainly driven by the absence of the second (private) fiber provider.

I find that the social planner's network does indeed improve total surplus: fiber consumers are 1.5% better off and total welfare increases by 1.4%. The municipal provider does not profit significantly more or less than the private fiber incumbent[47].

Finally, Figures 18 and 19 plot the changes in consumer surplus and market share, respectively, by technology type for different demographic groups. I find that municipal fiber benefits high-income consumers at the significant expense of low-income households, particularly those without kids. This is because the municipal network expands fiber access to lower-density neighborhoods where both per-household expansion costs and incomes are higher. On the other hand, consumer surplus from cable provision decreases for nearly all demographic groups (excluding unmarried, high-income households) due to lower quality plan offerings[48]; low-income households who are most price-sensitive are especially worse off. These results highlight the challenges of balancing the gains from new technology rollout against the intermediate losses to consumers of the legacy technology.

Local Loop Unbundling. In this counterfactual setting, I first consider a case where Boston directly adopts the UK's unbundling model: entrants have direct access to the last-mile network and pay a two-part tariff set by the government based on the incumbents' estimated long-run costs. In USD, this is roughly equivalent to a connection fee of $120 and bandwidth-invariant leasing fee of $100 per line. In my implementation, I assume that the local entrant, RCN, leases instead of building its own cable and fiber networks, and that there are no other entrants. Table 12 reports the impact on expansion, investment, prices, and speeds[49]. I find that under this unbundling scheme, fiber access decreases (by 6.5 tracts or 9,700 households on average each period) and roughly one-fifth of all network investment is cut across both access technologies—in response to free riding by entrants.

[47] Producer surplus from fiber increases by 0.1%.

[48] Figure 19 shows that access is not the primary driver as the share of households subscribed to cable is virtually unchanged.

[49] Detailed time-series trends can be found in Appendix A.

Average price per megabit falls by 2% because entrants are more efficient[50] and can offer more competitive prices. Average speeds decline by nearly 20% in response to a combination of effects triggered by lower fiber investment: the leasing fiber entrant is network constrained and unable to offer faster speeds; and the cable incumbent faces less speed competition and lowers its own plan speeds and investment.

Welfare analysis suggests that, in the long run, this policy is a net positive for all parties: consumer surplus increases 12%, producer surplus grows nearly 60%, and the total welfare in the market jumps 14%. However, all of the consumer gains arise from increased variety rather than improvements to access or plan quality. The entrants are able to leverage the incumbents' more extensive networks to service a larger share of households; plans characteristics themselves do not change significantly (see Figure 21 for details). Consequently, entrants earn significantly higher profits compared to the facilities-based competitive baseline. Incumbents, on the other hand, are worse off[51]. Fee-setting models based on cost depreciation, as the UK uses, are calibrated to maintain a level of 'reasonable' profit for incumbents, but omit a critical detail of the market: the strategic responses of firms. Naturally, this raises two questions: 1) how do connection and leasing fees affect firm strategies, and 2) does there exist a better tariff structure for the Boston market?

Figures 22 and 23 plot the impact of connection and leasing fees (holding the other fixed at the UK rate) on different outcomes of interest. Intuitively, higher connection fees disincentivize new entry while leasing fees affect the timing of entry. I find that there is a U-shaped relationship between leasing fees and fiber expansion (as measured by tracts and households with access). At around \$90 per line per month, the leasing fee is sufficiently high to overcome the losses from free-riding and the fiber incumbent begins to expand more rapidly. For leasing fees less than this tipping point, the relationship is reversed—expansion decreases because entrants connect in fewer markets. This trend is exactly mirrored in

[50]Entrants differ from incumbents by offered prices and speeds (arising from different marginal costs and stage-game optimization objectives) as well as firm quality term capturing intangibles such as customer service, reputation, etc.

[51]Incumbents' producer surplus falls by 2.3%.

	Baseline	UK Policy	% Delta
Access			
Tracts	63.3	56.8	-10.3%
Households	99.7	90.0	-9.7%
Investment			
Fiber	112.9	92.5	-18.0%
Cable	123.6	98.8	-20.1%
Avg. Speeds	119.6	97.3	-18.6%
Avg. Price/mb	20.4	20.0	-1.7%
Producer Surplus			
Total	19728.7	23973.1	+17.7%
Incumbents	11981.3	11712.0	-2.3%
Entrants	7747.4	12261.1	+58.3%
Consumer Surplus	51872.8	59117.1	+12.3%
Total Surplus	71601.4	83090.2	+13.8%

Table 12: Summary of market outcomes with and without unbundling under the UK benchmark policy.

entrant expansion outcomes. On the other hand, there is a weak negative correlation between leasing fees and investment. This is because entrants are more efficient, so higher capacities make entrants more competitive; this effect is compounded by the number of markets that the entrant competes in. The same is true of the relationship between connection fees and incumbent investment. Correlations between connection fees and entry outcomes for the fiber incumbent and both entrants are much more spurious. Incumbent entry weakly increases and entrant expansion weakly decreases as the connection fee grows.

Next, I evaluate whether there exists a two-part tariff that incentivizes entry and competition, benefits consumers, and maintains incumbents' total profits—in other words, a Pareto improvement over the status quo. I test this hypothesis by computing counterfactual outcomes for combinations of connection and leasing fees on a grid between \$0 and \$10,000. I then fit a spline on the outcomes of interest and evaluate whether there exists a point within this region that satisfies all conditions.

Ultimately, I find that there does indeed exist a non-empty region of the fee space that achieves all desired outcomes: a nominal \$15 connection fee and \$160 leasing fee within this set maximizes total surplus. Under this scheme, there is a small decline in fiber expansion

	Baseline	Pareto	% Delta
Access			
Tracts	63.3	57.2	-9.5%
Households	99.7	90.5	-9.3%
Investment			
Fiber	112.9	109.4	-3.0%
Cable	123.6	111.4	-9.9%
Avg. Speeds	119.6	100.7	-15.8%
Avg. Price/mb	20.4	20.0	-1.7%
Producer Surplus			
Total	19728.7	25120.9	+27.3%
Incumbents	11981.3	13248.9	+10.6%
Entrants	7747.4	11872.0	+53.2%
Consumer Surplus	51872.8	59279.5	+14.3%
Total Surplus	71601.4	84400.3	+17.9%

Table 13: Summary of market outcomes with and without unbundling under the Pareto improvement fee structure.

and investment; however, both increase relative to outcomes under the UK benchmark. Figures 25 and 24 plot speed and price responses, respectively. On the consumer side, surplus grows by 14%; subscribers of cable and fiber are both better off; and low income households, particularly those with children, benefit the most from the changes in fiber[52]. Entrants and incumbents are also both better off, earning 53% and 11% more in profits, respectively. Total welfare expands by 18% compared to the facilities-based baseline. This policy trades $4 million in entrant surplus for $15 million in incumbent surplus compared to the UK policy. Notably, due to the large inefficiencies of simultaneously building two new networks, fiber providers benefit more from this policy than cable providers.

7 Conclusion

Competition—access and quality—in broadband has received growing policy interest in the US in recent years. The market remains heavily concentrated despite rapid technological change due to high fixed costs of entry and returns to scale from network interactions.

[52]See Figure 18 for a more detailed decomposition of demographic differences in outcomes.

Design of efficient policies to address entry and investment requires an understanding of providers' static and dynamic incentives in this complex environment.

In this paper, I develop a novel, reinforcement learning-based full solution approach to solve the high-dimensional spatial competition game in broadband. First, I document strategic channels that arise in the environment: 1) households have heterogeneous tastes for internet speed and price, which firms can cater towards by offering different menus of plans; 2) there is evidence of early-mover advantages to deter entry from same-technology competitors; and 3) network span affects both expansion and stage game incentives. Then, I investigate the static (affordability and quality) and dynamic (access and investment) equilibrium outcomes of counterfactual technological and policy environments. I find that rollout of new technology can have unintended negative consequences for customers of legacy technology as those providers adjust their pricing strategies. Moreover, while unbundling significantly benefits consumers by incentivizing new entry and competition, calibration of fees is necessary to incentivize both participation from potential entrants (to ensure leasing is more profitable than building) as well as incumbents (to ensure profits from leasing outweigh losses from static competition). These results suggest that a cost-depreciation model that omits firms' strategic responses may not yield a Pareto improvement over facilities-based competition in the long run.

This analysis omits some characteristics of the industry for tractability. I do not allow for provider exit, model a firm's first entry decision (i.e. setting up a regional hub and choosing a subset of markets to service first), nor account for the potential impact of congestion, which may be a binding constraint over longer horizons without further investment or technological innovation. Another interesting avenue of future research involves endogenizing the technological innovation process, which I have taken as exogenous and readily available in the current model. Given that transmission protocols behave similarly to a common good, changes to one firm's investment incentives may have market-wide implications for the rate of innovation.

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Appendices

A Figures

β	Coefficient	Interaction with Demographics			
		Married	Kids	Income	Income^2
Price	-9.142***	1.860***	-0.358	2.614**	0.507
	(1.037)	(0.674)	(0.474)	(1.113)	(0.313)
Download	2.049***	0.006	-0.075	-0.713***	-0.069
	(0.192)	(0.132)	(0.096)	(0.211)	(0.061)
Capped	-0.242***				
	(0.040)				

Table 14: Estimates from demand model specification with technology, period, firm, and city FEs; limited to plans with market share $\geq 0.1\%$ across all three metropolitan areas.

Table 15: Estimates from demand model specification with technology, period, and firm FEs; limited to plans with market share $\geq 0.1\%$ in Boston.

β	Coefficient	Interaction with Demographics			
		Married	Kids	Income	Income^2
Price	-19.243***	4.468	-4.493***	37.186***	-10.761***
	(3.656)	(6.426)	(1.441)	(8.290)	(1.956)
Download	3.584***	-1.105	0.983***	-7.350***	2.164***
	(0.618)	(1.292)	(0.293)	(1.536)	(0.368)
Upload	1.423***				
	(0.090)				
Capped	0.143				
	(0.113)				

Table 16: Estimates from demand model specification with technology, period, and firm FEs; limited to plans with market share $\geq 0.1\%$ in Boston.

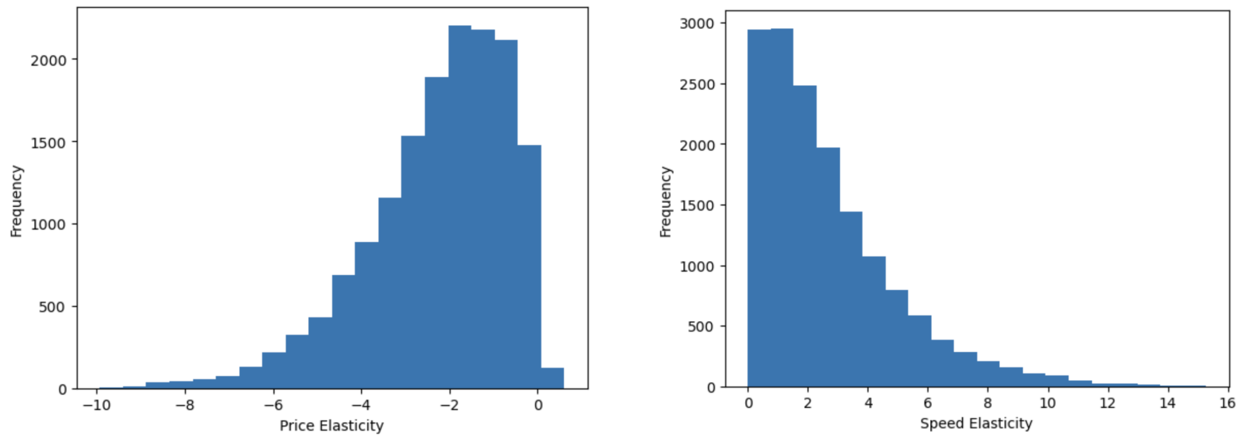


Figure 13: Densities of elasticities for price (left) and download speed (right) from the model specification without upload speeds.

	Comcast	HNS	RCN	Verizon	Viasat
Comcast	0.721	0.015	0.141	0.114	0.010
HNS	0.312	0.581	0.054	0.048	0.006
RCN	0.722	0.014	0.190	0.066	0.008
Verizon	0.636	0.063	0.039	0.271	-0.009
Viasat	0.394	0.009	0.069	0.062	0.466

Table 17: Firm-level price diversion ratios (averaged across markets); diagonal captures the outside-share.

	Cable	DSL	FTTH	Satellite
Cable	0.723	0.047	0.217	0.014
DSL	0.738	0.699	-0.486	0.050
FTTH	0.662	-0.012	0.335	0.015
Satellite	0.450	0.035	0.126	0.390

Table 18: Technology-level price diversion ratios (averaged across markets); diagonal captures the outside-share.

	Comcast	HNS	RCN	Verizon	Viasat
Comcast	0.685	0.017	0.149	0.138	0.010
HNS	0.389	0.373	0.065	0.168	0.005
RCN	0.692	0.013	0.186	0.103	0.007
Verizon	0.803	0.016	-0.117	0.288	0.010
Viasat	0.429	0.009	0.075	0.074	0.412

Table 19: Firm-level download speed diversion ratios (averaged across markets); diagonal captures the outside-share.

	Cable	DSL	FTTH	Satellite
Cable	0.637	0.097	0.252	0.014
DSL	0.542	0.343	0.103	0.012
FTTH	0.646	0.038	0.303	0.013
Satellite	0.458	0.143	0.132	0.267

Table 20: Technology-level download speed diversion ratios (averaged across markets); diagonal captures the outside-share.

	Active Tracts			Household Access (in 1000s)		
Period	Counterfactual	Simulated	% Delta	Counterfactual	Simulated	% Delta
2015Q2	4.00	4.00	0.00	8.88	8.88	0.000
2015Q4	10.92	12.72	-0.142	19.53	22.01	-0.113
2016Q2	23.36	24.68	-0.053	38.19	39.42	-0.031
2016Q4	38.48	40.52	-0.050	60.74	62.71	-0.031
2017Q2	59.36	59.96	-0.010	92.00	92.11	-0.001
2017Q4	74.48	76.32	-0.024	115.09	117.96	-0.024
2018Q2	91.36	93.04	-0.018	141.13	144.34	-0.022
2018Q4	107.60	110.84	-0.029	169.02	173.85	-0.028
2019Q2	121.80	125.40	-0.029	192.82	197.39	-0.023
2019Q4	135.76	139.28	-0.025	215.68	221.22	-0.025

Table 21: Comparison of Verizon fiber availability over time with and without RCN competition.

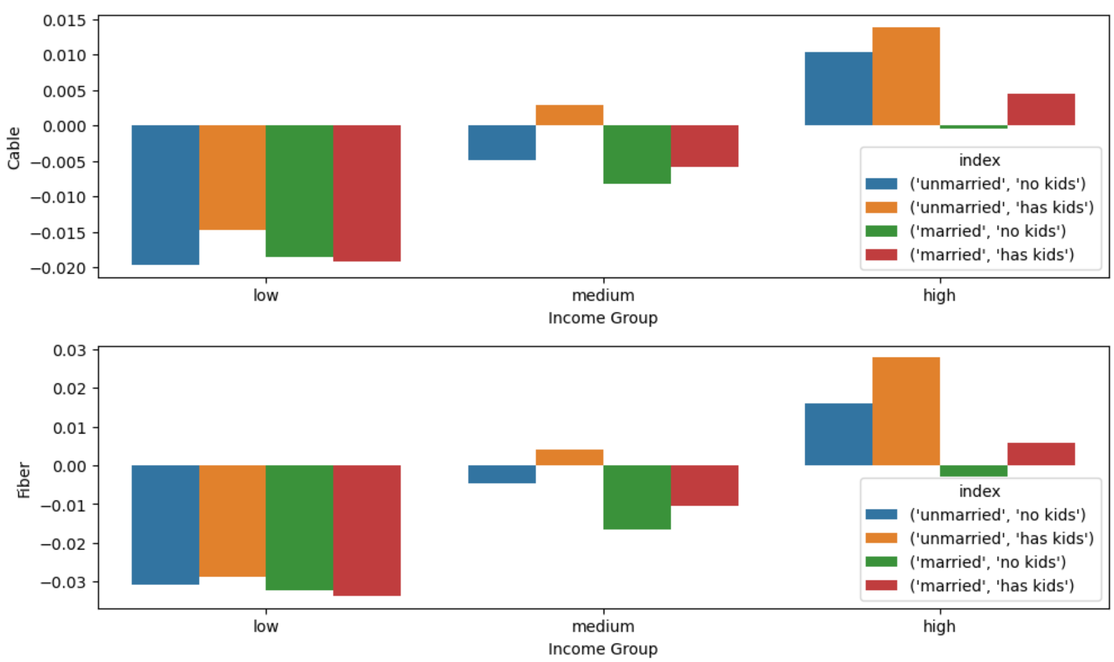


Figure 14: Change in consumer surplus due to static responses to a ban on same-technology competition.

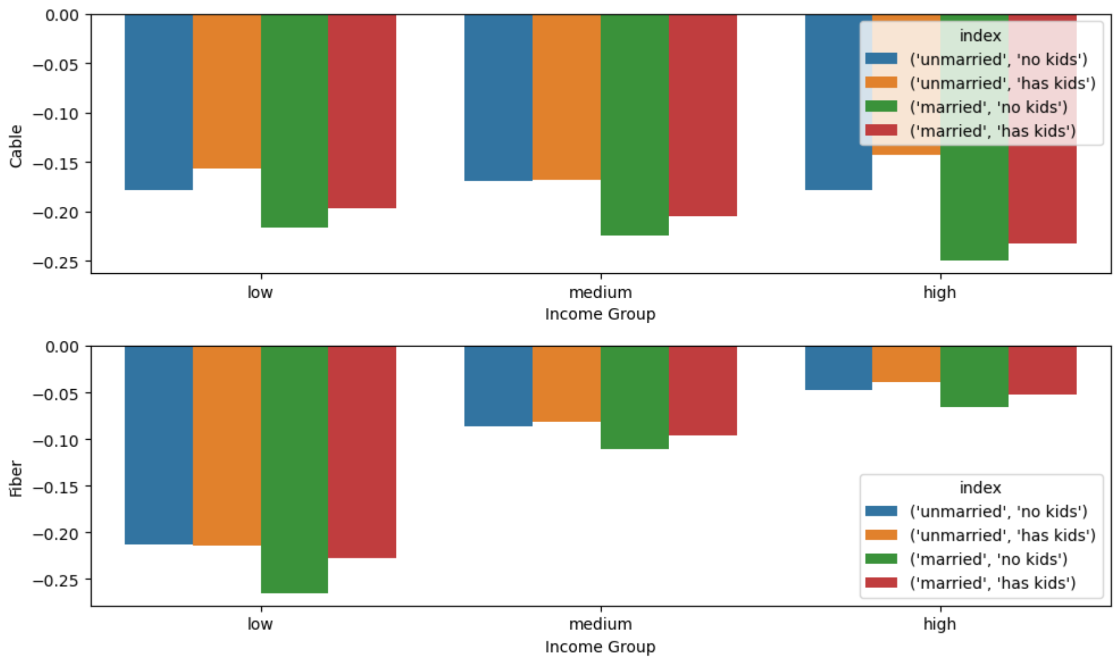


Figure 15: Change in consumer surplus due to dynamic responses to a ban on same-technology competition.

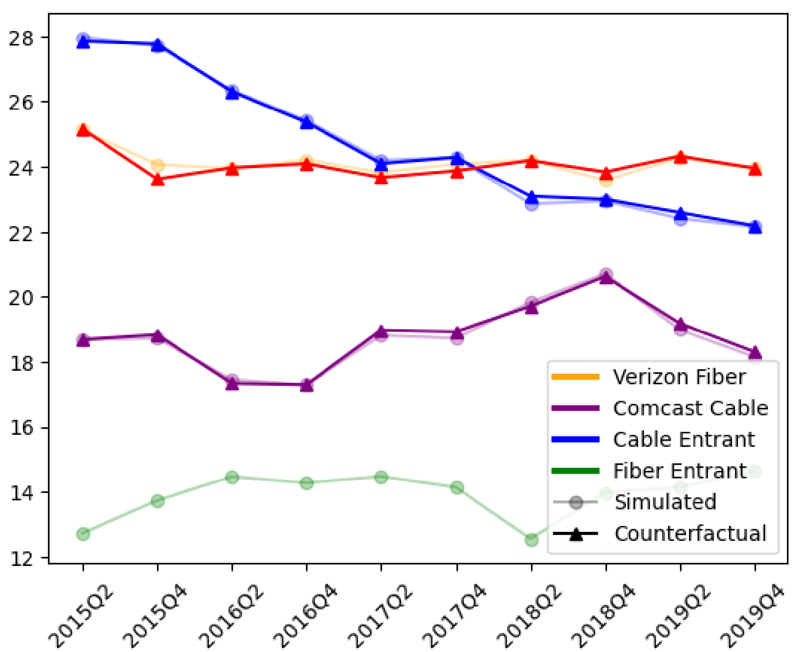


Figure 16: Plan price per megabit with and without municipal fiber provider.

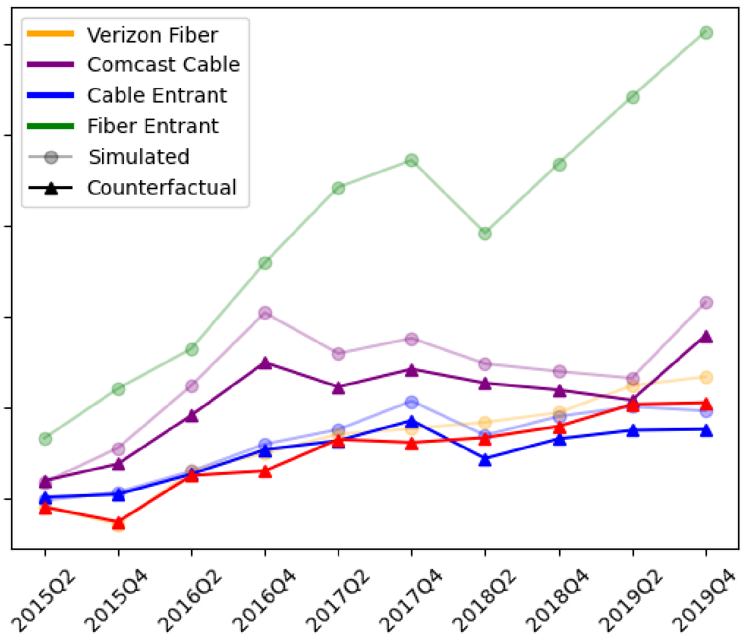


Figure 17: Plan download speeds with and without municipal fiber provider.

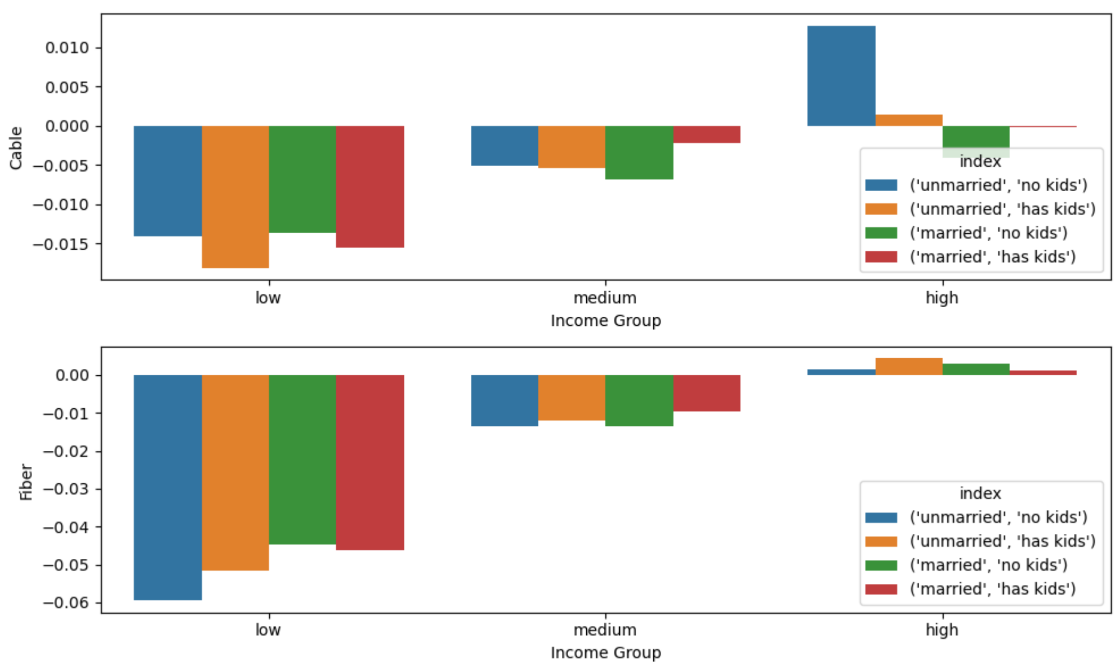


Figure 18: Change in consumer surplus due to municipal fiber provision by demographic group, denoted (married, kids, income).

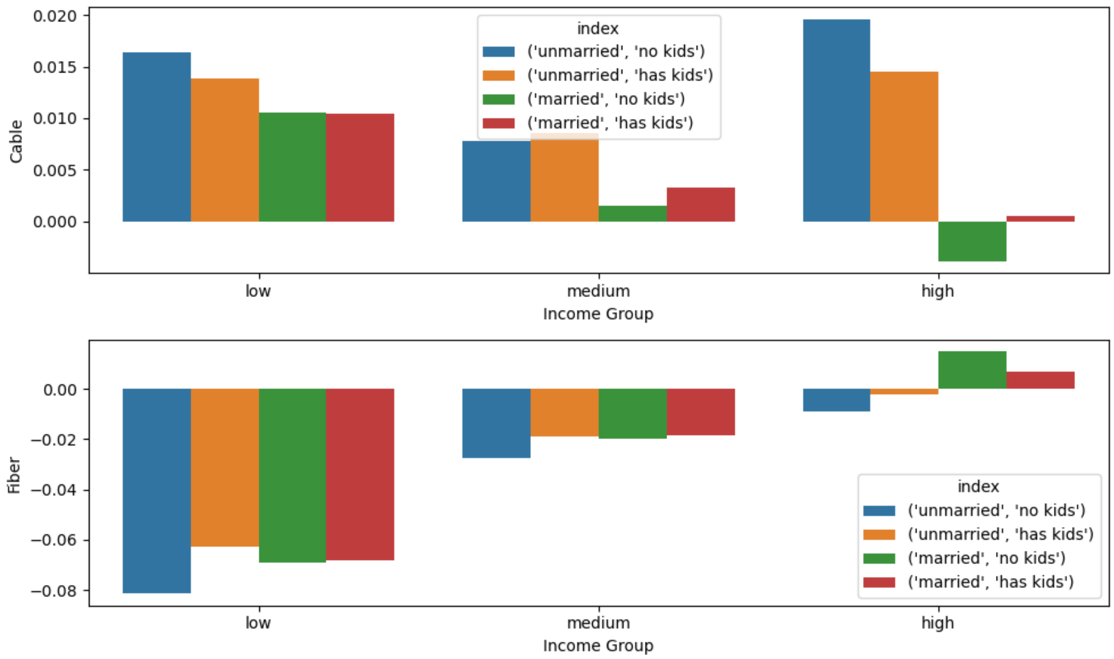


Figure 19: Change in market share due to municipal fiber provision by demographic group, denoted (married, kids, income).

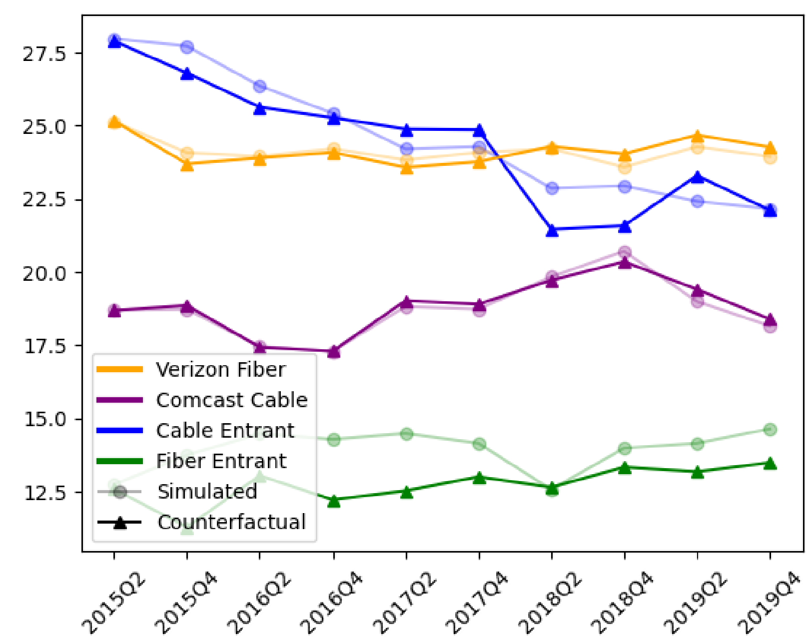


Figure 20: Plan price per megabit with and without unbundling under the UK benchmark policy.

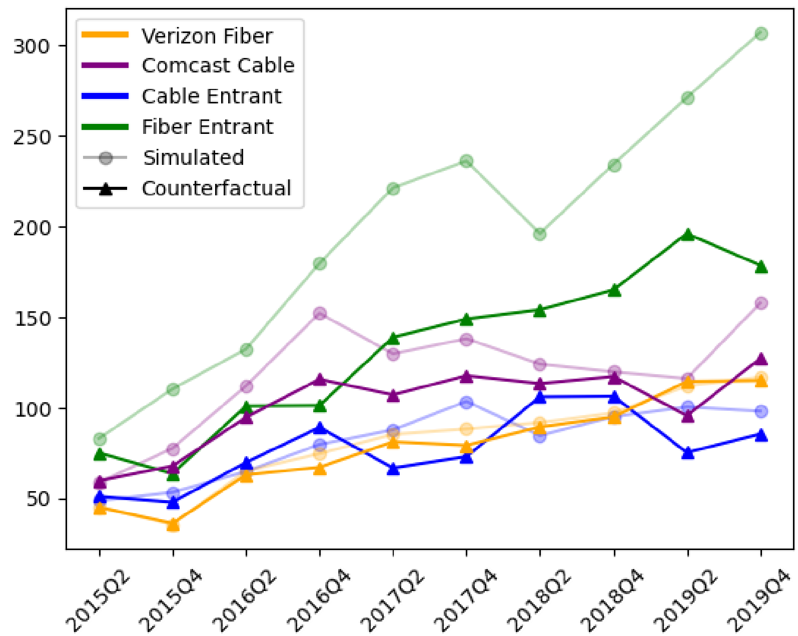
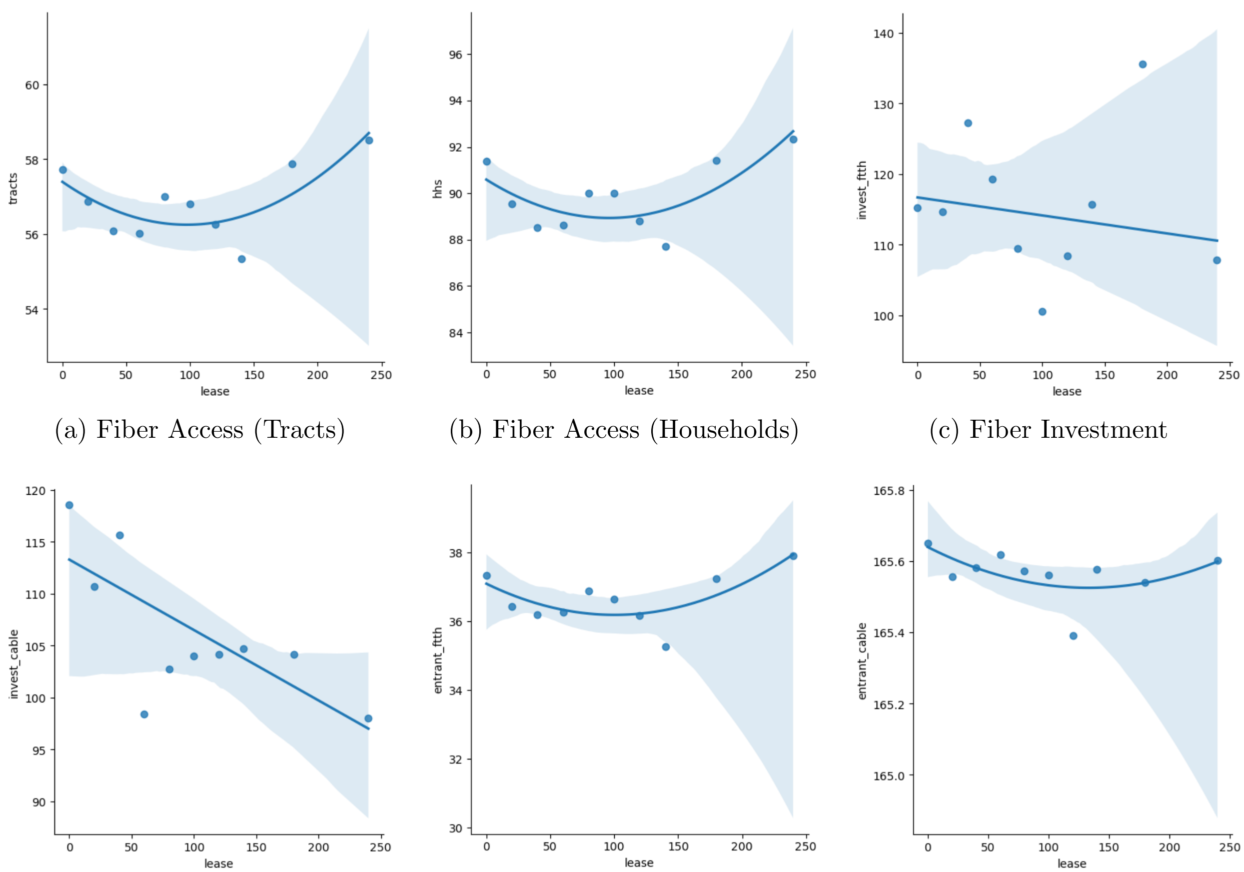


Figure 21: Plan download speeds with and without unbundling under the UK benchmark policy.



(a) Fiber Access (Tracts)

(b) Fiber Access (Households)

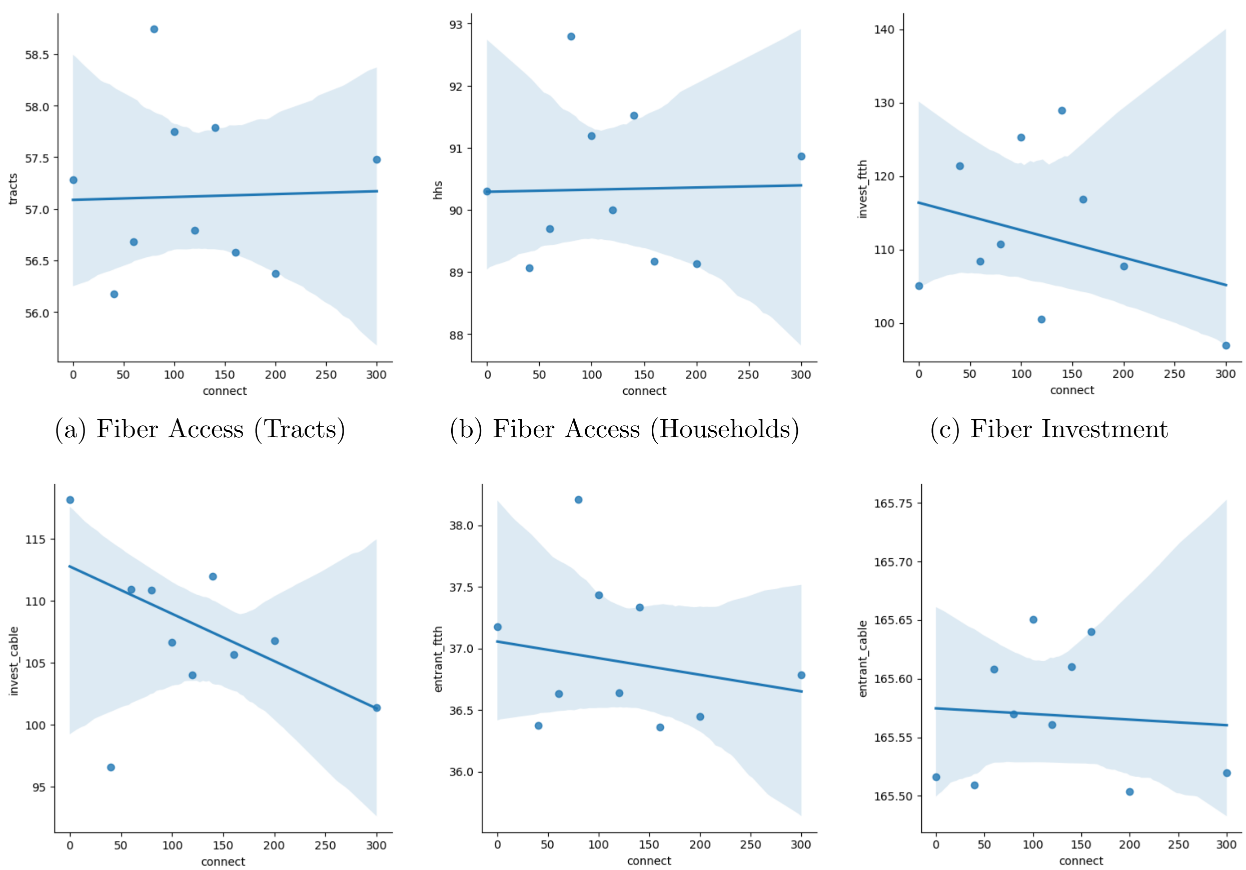
(c) Fiber Investment

(d) Cable Investment

(e) Entrant Expansion (Fiber)

(f) Entrant Expansion (Cable)

Figure 22: Impact of leasing fees on market outcomes (holding connection fees fixed).



(a) Fiber Access (Tracts) (b) Fiber Access (Households) (c) Fiber Investment

(d) Cable Investment (e) Entrant Expansion (Fiber) (f) Entrant Expansion (Cable)

Figure 23: Impact of connection fees on market outcomes (holding leasing fees fixed).

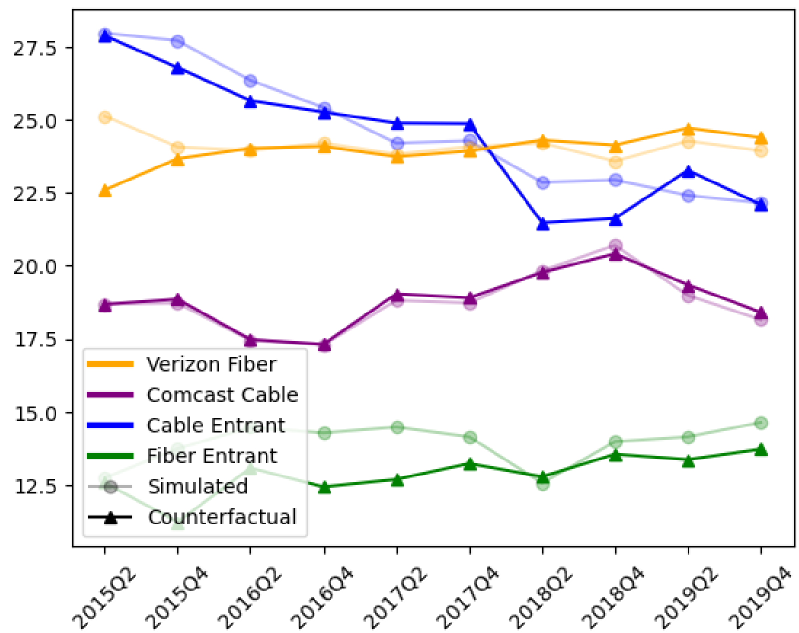


Figure 24: Plan price per megabit with and without unbundling under the pareto improvement policy.

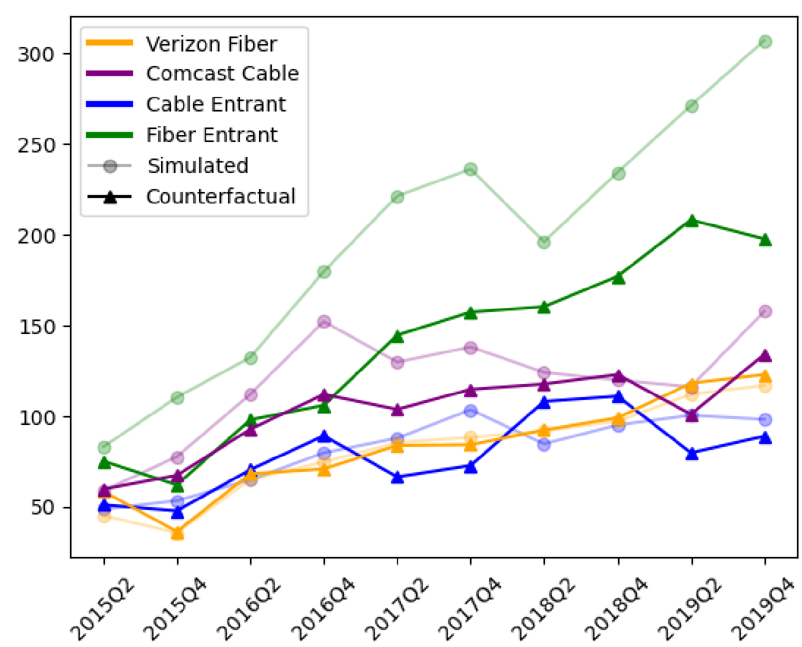


Figure 25: Plan download speeds with and without unbundling under the pareto improvement policy.

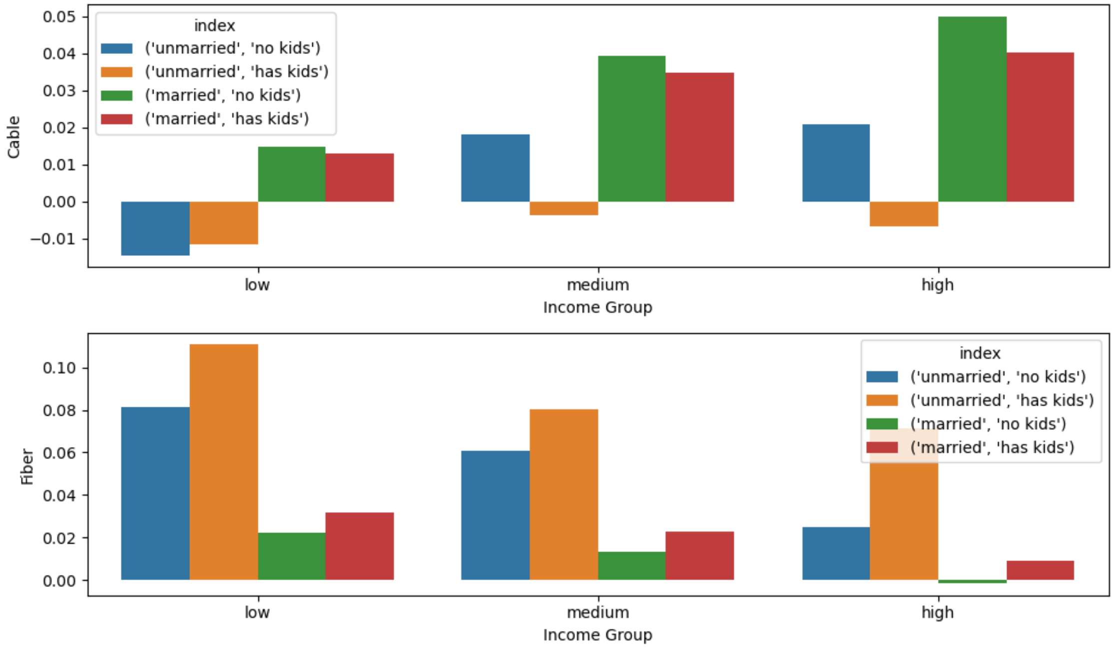


Figure 26: Change in consumer surplus under the pareto improvement policy by demographic group, denoted (married, kids, income).

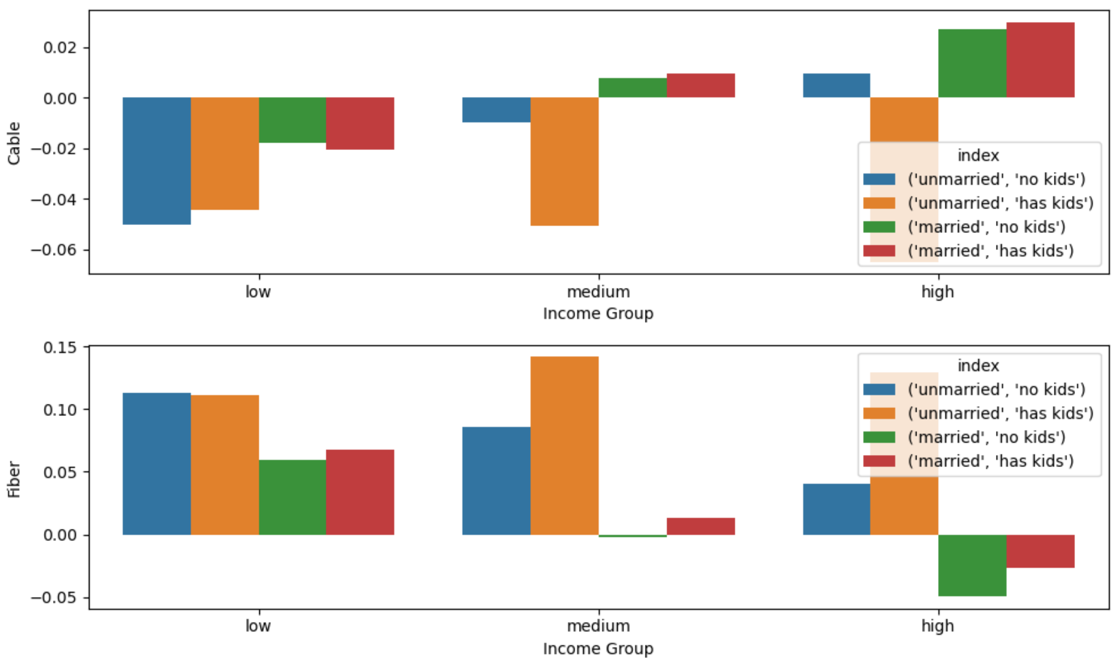


Figure 27: Change in market share under the pareto improvement policy by demographic group, denoted (married, kids, income).

B Plan Shares Algorithm

For each test obtained from Google's MLab database, I observe the time, ISP, geographic location (zip code), and measured download and upload speeds. Under the assumption that consumers are equally likely to test their internet speeds regardless of the plan they are subscribed to, I can leverage the density of test speeds to infer the distribution of consumers across plans in each zip, conditional on ISP. I also obtain plan share data from an anonymous ISP in 4 MSAs for each month between April and December 2016, inclusive.

The training set for the K-nearest neighbors (KNN) algorithm consists of 3rd-order polynomial transformations of the upload and download speeds advertised by the anonymous provider in each MSA (plus an additional dummy plan with zero speeds to isolate tests from network outages). I supplement this with the same transformation of average measured upload and download speeds from the MBA survey for consumers belonging to the same state. Because plan choices are observed for these households, inclusion of this dataset helps account for noise, congestion, etc. that causes measured speeds to differ from those advertised. I then evaluate the KNN algorithm on MLab speed tests conducted in zip codes belonging to each MSA and select the model that best matches the monthly shares of the plans from the anonymous ISP. The hyperparameters of the clustering algorithm are fine-tuned using 4-fold cross-validation (by omitting one MSA in each fold) to minimize overfitting. Model predicted versus observed plan shares (averaged over the 4 MSAs) in Figure 28 suggest the algorithm does a fairly good job at the prediction task.

Finally, I use this model to predict the plan shares by zip code for each ISP in the Boston market. For ISPs that offer multiple technology types, I pre-classify the speed tests by technology type using a random forest classifier trained on the reported MBA data with hyperparameters tuned using 5-fold cross-validation. This reduces the possibility of overlap between potential targets when two plans offer similar speeds. For some smaller ISPs, I don't observe any consumer units in the MBA dataset, so I train a separate model with only advertised plan speeds and use this to cluster the speed tests. An example of the raw density

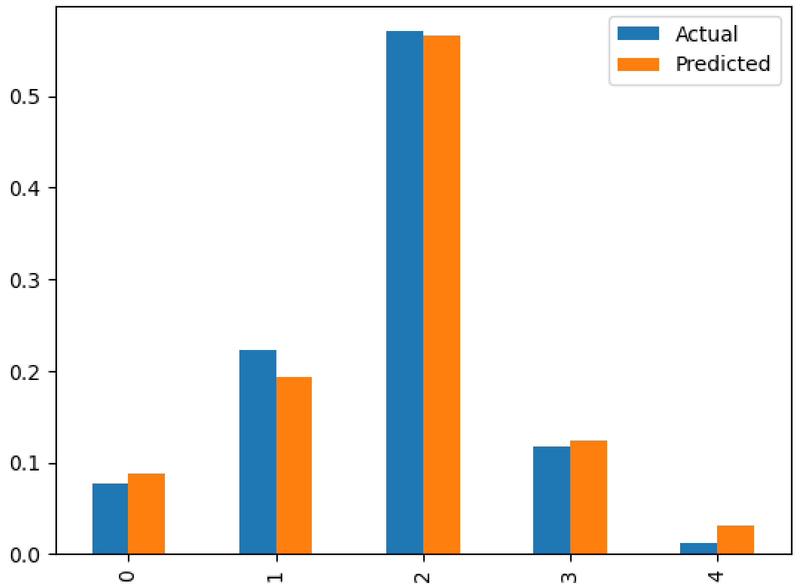


Figure 28: Fit of the KNN algorithm (averaged across MSAs).

of tests by upload and download speed compared to the density of matched tests (each color denotes a different plan; tests attributed to outages or noise are dropped) can be found in Figures 29 and 30, respectively.

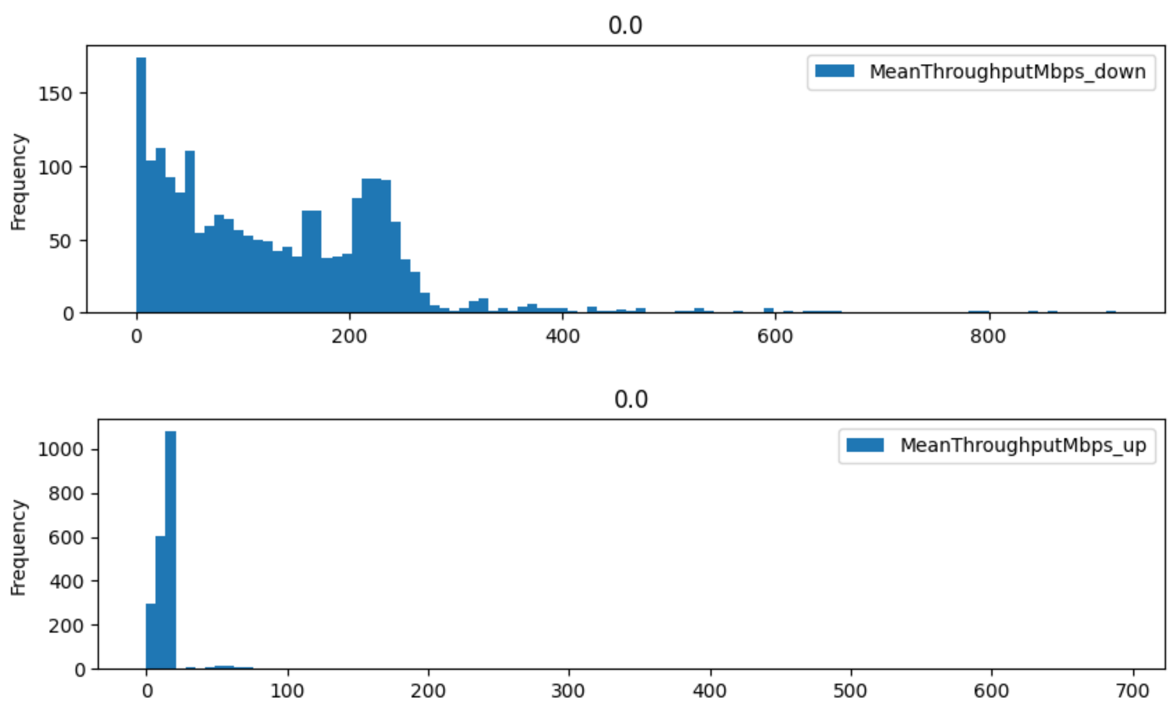


Figure 29: Raw download (top) and upload (bottom) speed distributions for RCN.

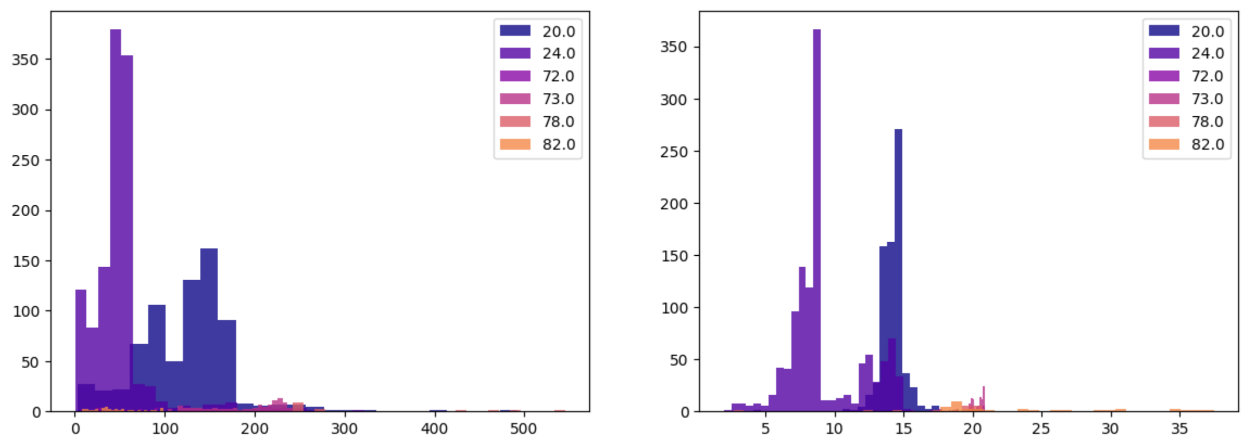


Figure 30: Matched download (left) and upload (right) speed distributions for RCN.

C Proof of EBE Conditions

Let $\mathbb{P}^*$ denote the CCPs under σ^*. Firm and time subscripts are omitted for notational clarity. By Lemma 1 of Arcidiacono and Miller (2011) (for the case of logit shocks):

$$
\begin{aligned}
Q^*(s,a) &= \Pi(s,a) + \beta\mathbb{E}[V^*(s')] \\
&= \Pi(s,a) + \beta\mathbb{E}[\max_{a'}\{Q^*(s',a') + \eta\epsilon(a')\}] \\
&= \Pi(s,a) + \beta\mathbb{E}[\eta(\gamma - \ln\mathbb{P}^*(s',\sigma^*(s'))) - \max_{a'} Q^*(s',a') + Q^*(s',\sigma^*(s'))] \\
&= \underbrace{\mathbb{E}[\Pi(s,a) + \beta(\eta\gamma + \eta\log\sum_{a'}\exp(Q^*(s',a')/\eta) - \max_{a'} Q^*(s',a'))]}_{r^*(s,a,s')} + \beta\mathbb{E}[Q^*(s',\sigma^*(s'))]
\end{aligned}
$$

which has the desired form in the definition of the EBE. Notice, however, that the reward function depends on the vector of next-state conditional values $Q^*(s_{ft+1},\cdot)$. In the computer science literature, this functional form is known as entropy-regularized reinforcement learning (Haarnoja et al. (2017), Neu et al. (2017), Geist et al. (2019)) and can be shown to satisfy the conditions in Fershtman and Pakes (2012) necessary for Q-learning algorithm convergence. Formally, Geist et al. (2019) show that if function $\phi : \mathbb{R}^{|\mathcal{A}|} \to \mathbb{R}$ is monotone and L-Lipschitz for $L \leq 1$, then the operator

$$(T_\phi Q)(s,a) = \Pi(s,a) + \beta\mathbb{E}[\phi(Q(s',\cdot))]$$

is a βL-contraction. Consequently, the Q-learning proof with modified update target $\Pi(s,a) + \beta\phi(Q(s',\cdot))$[53] still goes through. Let

$$\phi(x) = \eta\gamma + \eta\log\sum_{a'}\exp(x_{a'}/\eta)$$

[53] Instead of $\Pi(s,a) + \beta Q(s',\cdot)$.

This function is monotone and has Lipschitz constant $L = 1$ (Littman and Szepesvári (1996)). Thus, the target

$$\begin{aligned}\Pi(s,a) + \beta\phi(Q^*(s',\cdot)) &= \Pi(s,a) + \beta(\phi(Q^*(s',\cdot)) - \max_{a'} Q^*(s',a') + Q^*(s',\sigma^*(s'))) \\ &= r^*(s,a,s') + \beta Q^*(s',\sigma^*(s'))\end{aligned}$$

converges under standard Q-learning assumptions.

An alternative approach that reduces the computational complexity of the update rule is to use a 'softmax' approximation. Suppose $r(s,a) = \Pi(s,a) + \beta\eta(\gamma + c)$ for some constant c. The error in this approximation can be expressed as:

$$\begin{aligned}|r^*(s,a,s') - r(s,a)| &= |\beta(\eta \log \sum_{a'} \exp(Q^*(s',a')/\eta) - \max_{a'} Q^*(s',a') - \eta c)| \\ &= |\beta\eta(\log \sum_{a'} \exp((Q^*(s',a') - \max_{a''} Q^*(s',a''))/\eta) - c)|\end{aligned}$$

Let $\Delta(s') = \eta(\log \sum_{a'} \exp((Q^*(s',a') - \max_{a''} Q^*(s',a''))/\eta) - c)$ has uniform bounds

$$-\eta c \leq \Delta \leq \eta(\log |\mathcal{A}| - c)$$

The minimax choice $c = \log |\mathcal{A}|/2$ yields the tightest worst-case bounds:

$$|\beta \mathbb{E}[\Delta(s')]| \leq \beta\eta \log |\mathcal{A}|/2$$

In my estimation, I use this softmax approximation, where $|\mathcal{A}| = 2$.

D Marginal Cost Estimates

Figure (31) shows the density of estimated c_0 and c_1 marginal cost parameters separately for each of the cable and fiber firms (where RCN's cable and fiber services are treated as individual firms).

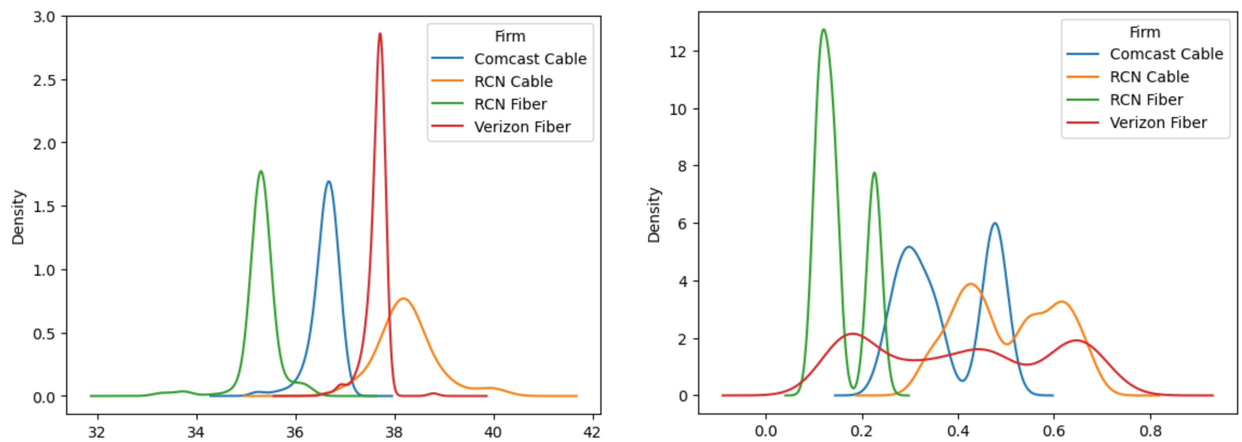


Figure 31: Left panel: c_0 estimates; right panel: c_1 estimates.

E Dynamic Estimation Algorithm

E.1 Adaptation for Continuous State Space

To account for the continuous nature of the state space, I adapt the kernel-based approach of Ormoneit and Sen (2002) for (5). Define the kernel operator:

$$k_{T^a,b}(s', s) = \phi(\frac{||s' - s||}{b}) / \sum_{(x,y) \in T^a} \phi(\frac{||x - s||}{b})$$

where ϕ is a univariate, non-negative mother kernel function (e.g. Gaussian, Epanechnikov, etc.), b is the bandwidth parameter that controls the degree of smoothing, and T^a is the set of all observed (x, y) transitions where action a was taken. The update rule can be rewritten as the weighted sum

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha \sum_{(x,y)\in T^a} k_{T^a,b}(x,s) \cdot (r(x,a) + \beta \max_{a'} Q(y,a'))$$

Intuitively, this approximates the value of taking action a in state s as a weighted sum of instances where action a was taken in states similar to s. Note that this expression is only ever evaluated at *observed* states, dramatically reducing the computational complexity of the algorithm for estimation. In solving for the counterfactual equilibria, I repeatedly simulate these T^a transition sets until both the transition path and value function estimates stabilize.

E.2 Algorithm Steps

The specific implementation I use is as follows:

1. Define a grid over the parameter space for θ_3.
2. For a given point θ_3 in this grid, initialize matrix $Q^0(s,a;\theta_3)$ for every observed state s and possible action $a \in \{0,1\}$
3. Pre-compute the kernel matrices for each action a and observed transition set T^a using an Epanechnikov kernel with density given by

$$\phi(u) = \frac{3}{4}(1-u/b)^2 \qquad \text{for } |u| \leq b$$

   and bandwidth equal to the distance of the $n(a)$-th closest action state where $n(a)$ is the minimum of 10 or the total number of states in which action a was observed.
4. For value function iteration $k = 0,1,2,3...$:
   (a) For every observed state s, select action a according to the locally ϵ-greedy policy[54] and compute the updated value $Q^{k+1}(s,a;\theta_3)$ according to (5). If action a is never

[54] With probability $1-\epsilon$, take the action that maximizes $Q(s,a)$; with probability ϵ, take a random action.

observed in state s, apply the kernel approximation of the update rule instead[55].

(b) If $||Q^{k+1} - Q^k|| < 1e - 8$ the algorithm has converged

(c) Otherwise, update $\alpha = 1/k^{0.9}, \epsilon = \max\{0.9^k, 0.01\}$ and repeat.

5. Evaluate the objective (6) on every (s, a) observed in the data. Update θ_3 and repeat until minimized.

E.3 Toy Model Verification

I numerically verify the convergence of the proposed reinforcement learning algorithm to the optimal policy on the toy model in Section E.3. I construct an artificial dataset of firm actions and rewards based on the initial network illustrated in Figure 5; in this low-dimensional setting, the full solution is computationally tractable using an iterative approach, which can then be compared against the algorithm output.

Suppose the state space is given by

$$s = \{s_m\}_m \qquad s_m = (r_m, d_m, adj_m)$$

where d is the minimum distance as defined in the full game and adj_m is the number of inactive markets adjacent to market m. The latter variable is included to further differentiate between states and ensure that the observed (optimal) policy is deterministic when a state is repeatedly visited[56]. The initial state in Figure 5 is represented as:

$$s_0 = \{(0, 0, 2), (1, 3, 2), (0, 2, 3), (2, -1, 3), (4, -1, 2)\}$$

[55]Note that in contrast to more standard Q-learning update rules which focus on a single state (usually for a single agent problem), this step applies to an action in every single observed state. Transitions are computed assuming all other firms played optimally (i.e. followed their observed actions in that state). This strikes a balance between online (exploring and interacting with the environment) and offline (relying on observed history of states and actions) learning, significantly accelerating convergence.

[56]This is not a concern in the full game where the state space is sufficiently high dimensional and observations are limited.

The action space is given by $\{0,1\}^6$.

The per-period action-specific reward function is defined over the space of state and action vectors:

$$u(s, a; \gamma) = \sum_m r_m \cdot \mathbb{1}_{d_m=0} + \gamma' a$$

and the state transitions deterministically according to $T : \mathcal{S} \times \mathcal{A} \to \mathcal{S}$:

$$\begin{aligned} T(r_m) &= \min\{r_m + \mathbb{1}_{upgrade}, 5\} \\ T(d_m) &= \min_{n \in A_m} \{d_{mn} | d_n = 0\} \\ T(adj_m) &= \sum_{n \in A_m} \mathbb{1}_{d'_n \neq 0} \end{aligned}$$

where A_m denotes the set of tracts adjacent to m.

Following Rust (1987), define the value and expected value functions:

$$\begin{aligned} V(s) &= \max_a \{u(s,a) + \epsilon(a) + \beta \mathbb{E}[V(s')|s,a]\} \\ EV(s,a) &= \mathbb{E}_{s',\epsilon}[\log \sum_{a'} \exp(u(s',a') + \beta EV(s',a'))] \end{aligned} \tag{7}$$

Rust (1987) proves that (7) is a contraction mapping and, as such, EV can be estimated by iterating until convergence. I then back out the optimal policy:

$$\sigma(s) = \arg\max_a \{u(s,a) + \epsilon(a) + \beta EV(s,a)\}$$

This policy function can be used to generate a dataset of state-action pairs to be fed into the reinforcement learning algorithm.

The steps of the procedure are as follows:

1. Set $\gamma = (1, 3, 60)$

γ_0	γ_1	γ_2	Likelihood
1	3	80	145.662325
1	1	70	147.959118
1	**3**	**60**	**148.894803**
1	1	60	150.720915
1	4	100	151.845203
1	6	100	155.981831
1	8	80	157.271577
2	5	100	157.562729
1	7	70	158.338578
1	7	40	158.613391

Table 22: Toy model cost estimates (true γ in bold).

2. Generate a matrix of all reachable states from s_0, $\mathcal{S}_{estim} \subseteq \mathcal{S}$, and all allowable action vectors. Set all values to 0 and denote this matrix EV^0

3. For iteration $K = 0, 1, 2, 3...$

   (a) Compute $EV^k(s, a)$ according to (7) for all s, a pairs

   (b) If $\max_{s,a} |EV^{k+1}(s, a) - EV^k(s, a)| < 1e - 8$, continue to next step.

4. Compute $\sigma(s)$ for a random sample of states $s \in \mathcal{S}_{test} \subseteq \mathcal{S}_{estim}$; this is the artificial training dataset of 'observed' optimal actions.

5. Run the reinforcement learning algorithm on $\{(s, \sigma(s)) \mid s \in \mathcal{S}_{test}\}$ to obtain a parameter estimate $\hat{\gamma}$ and corresponding likelihood.

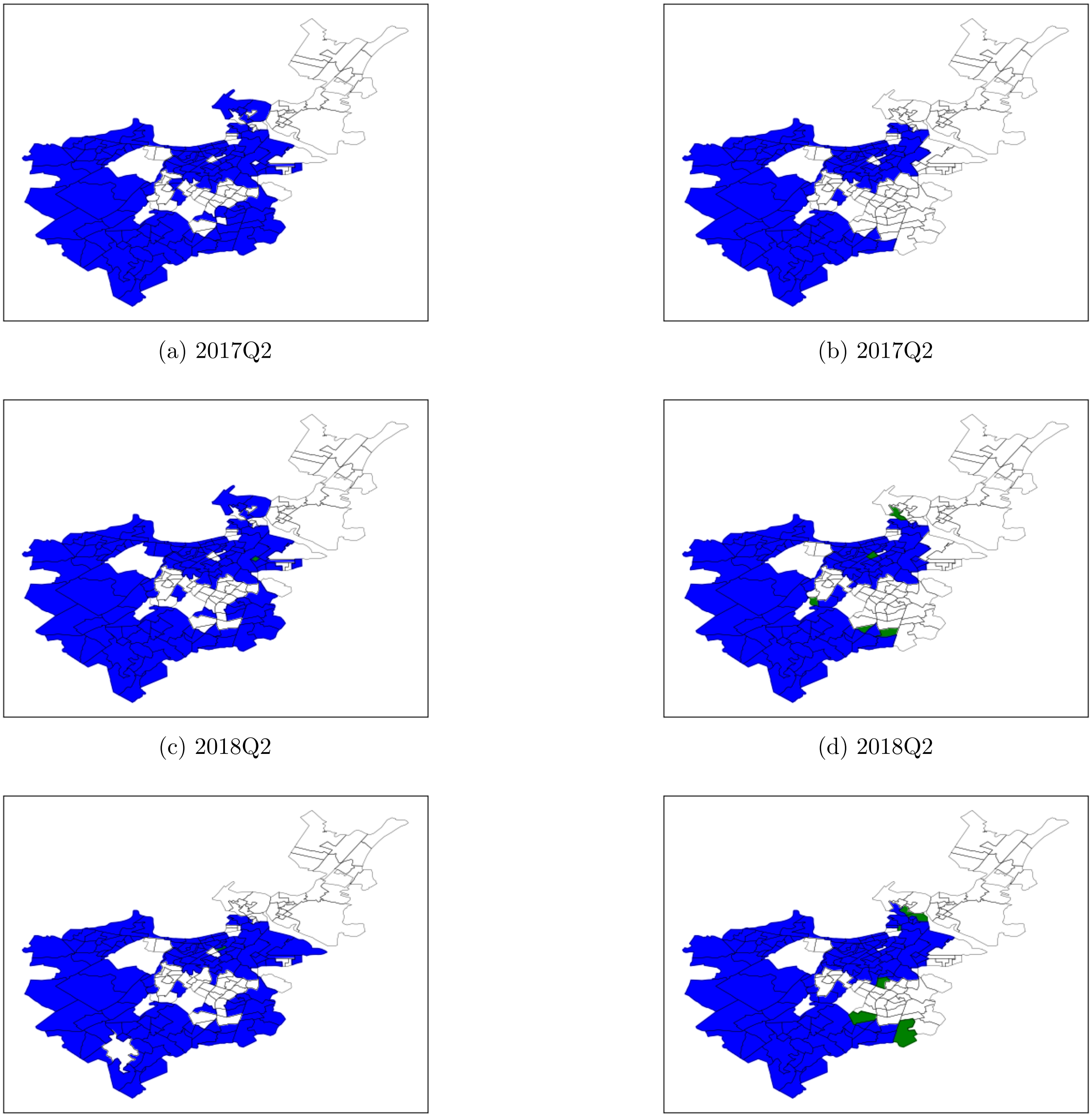
6. Verify that γ is in the neighborhood of $\hat{\gamma}$[57].

Table 22 shows the top parameter vectors from the grid search process ranked by their computed (negative log) likelihoods. The true γ_0 is within this set.

[57]Note that γ may not be point identified as many parameter vectors in the neighborhood of γ generate similar if not identical policy functions. To mitigate this issue, I also compute the optimal policies implied by nearby γs and include the states/paths where these policies differ in the training set $\mathcal{S}_{test}$

E.4 Additional Model Fit Metrics

Figures 32 and 23 illustrate the model-predicted expansion behavior of RCN.



(a) 2017Q2 (b) 2017Q2

(c) 2018Q2 (d) 2018Q2

(e) 2019Q2 (f) 2019Q2

Figure 32: Observed (left) vs predicted (right) expansion of RCN across Boston census tracts. Blue indicates services offered in previous period; green indicates new expansion.

Period	Sim. Active	Std. Error	Obs. Active
2015Q2	84.00	0.00	84
2015Q4	87.00	1.63	91
2016Q2	88.68	2.10	95
2016Q4	90.76	2.54	122
2017Q2	93.68	3.48	124
2017Q4	96.48	3.55	123
2018Q2	99.36	4.00	122
2018Q4	101.52	4.06	120
2019Q2	104.56	5.37	115
2019Q4	107.16	6.19	116

Table 23: Mean predicted versus observed number of active tracts per period for RCN.

F Counterfactual Solution Method

I begin by updating the state variables and profit functions to account for the changes in each environment:

No Same-Technology Competition. I remove RCN cable and fiber from all tracts and periods, and subtract their contributions from relevant state variables—$\bar{u}^n_{-ft}, nn^m_t, \max b_{-ft}$—accordingly.

Municipal Broadband. I remove RCN fiber from all tracts and periods and replace Verizon fiber with the social planner (existing infrastructure, costs, and demand-relevant variables, such as unobserved quality, remain unchanged). I also modify the firm's value function (and algorithm updates) to include consumer surplus from fiber

$$CS \approx -\frac{1}{\beta_p + \sigma_p^y \bar{y}} \log \sum_j \exp(\delta_j + \bar{\mu}_j)$$

in addition to flow profits. Here, $\bar{y}$ and $\bar{\mu}_j$ denote the weighted average of consumer demographics and indirect utility from product j, respectively, in a single market (i.e. $\int y_i dF_i$ and $\int \mu_{ij} dF_i$).

Local Loop Unbundling. To capture this environment, state transitions and flow profits must be updated. Let τ_0 and τ_1 denote the (one-time) connection and (per-period) leasing fees, respectively. I do not model the choice between leasing versus building and assume that all entrants default to leasing. For every potential entrant, d^m, the minimum build distance, is set to 1 for every market the incumbent has entered. The entry cost paid by entrants is equivalent to $t_0 \cdot lines$ (where $lines$ equals $pop/128$ and $pop/500$ fiber and cable networks, respectively), and transferred to the incumbent that owns the network. When the fiber incumbent expands into a new market, that market is also 'unlocked' and open for expansion for the leasing firms. Moreover, when either incumbent upgrades their network, the maximum capacity for all firms offering service on the network increases deterministically; entrants are not allowed to upgrade the incumbents' infrastructure. Every period, the entrants pay to the incumbents leasing fees equivalent to $\tau_1 \cdot lines$ in every market where they offer services.

Firms are differentiated in the stage game by their marginal costs. For new potential entrants, I assume that the distribution of estimated costs constitute the bounds on the support of the distributions from which c_0 and c_1 are (randomly) drawn for each observed firm. I assume uniform distributions and sample c_0 and c_1 marginal costs for potential entrants. In preliminary counterfactual analysis, I limit to just a single potential entrant for each incumbent; if a potential entrant does not enter any markets in a period, I assume it 'died' and draw new marginal costs for a new potential entrant the next period.

Algorithm. The steps for computing counterfactual equilibria are as follows:

1. Initialize D_0 to all states in the counterfactual dataset.

2. For $n = 0, ..., N$ resets:

   (a) Create sets S_0 and A_0 to store simulated paths.

   (b) Initialize Q^0 for all states in D_n and all actions $a \in \{0, 1\}$.

(c) For $k = 0, ..., K_n$ iterations[58]:

i. Set s_0 to the state in the first period in D_n.

ii. For $t = 0, ...T - 1$ periods[59]:

A. Compute prices and speeds for incumbents according to the stage game.

B. For each firm, take a random draw $\epsilon \sim Unif([0, 1])$ for every market, compute the CCPs $\mathbb{P}^k$ for each action in the current state, and take corresponding actions in each market m depending on whether $\epsilon_m \lesseqgtr \mathbb{P}^k(a = 1|s_t^m)$.

C. Compute flow profits based on plan revenues and action costs.

D. Store current states in S_k; store actions in A_k; update the state s_{t+1} according to all firms' actions.

iii. Update Q^{k+1} according to (5), treating S_k and A_k as observations of the globally optimal policy.

(d) Update $D_{n+1} = S_K$, the last simulated path.

[58] Ensure that K_N is sufficiently large for the final Q^{K_N} to converge.

[59] I use T equal to the number of observed periods in my data to avoid modeling the transitions of exogenous state variables.